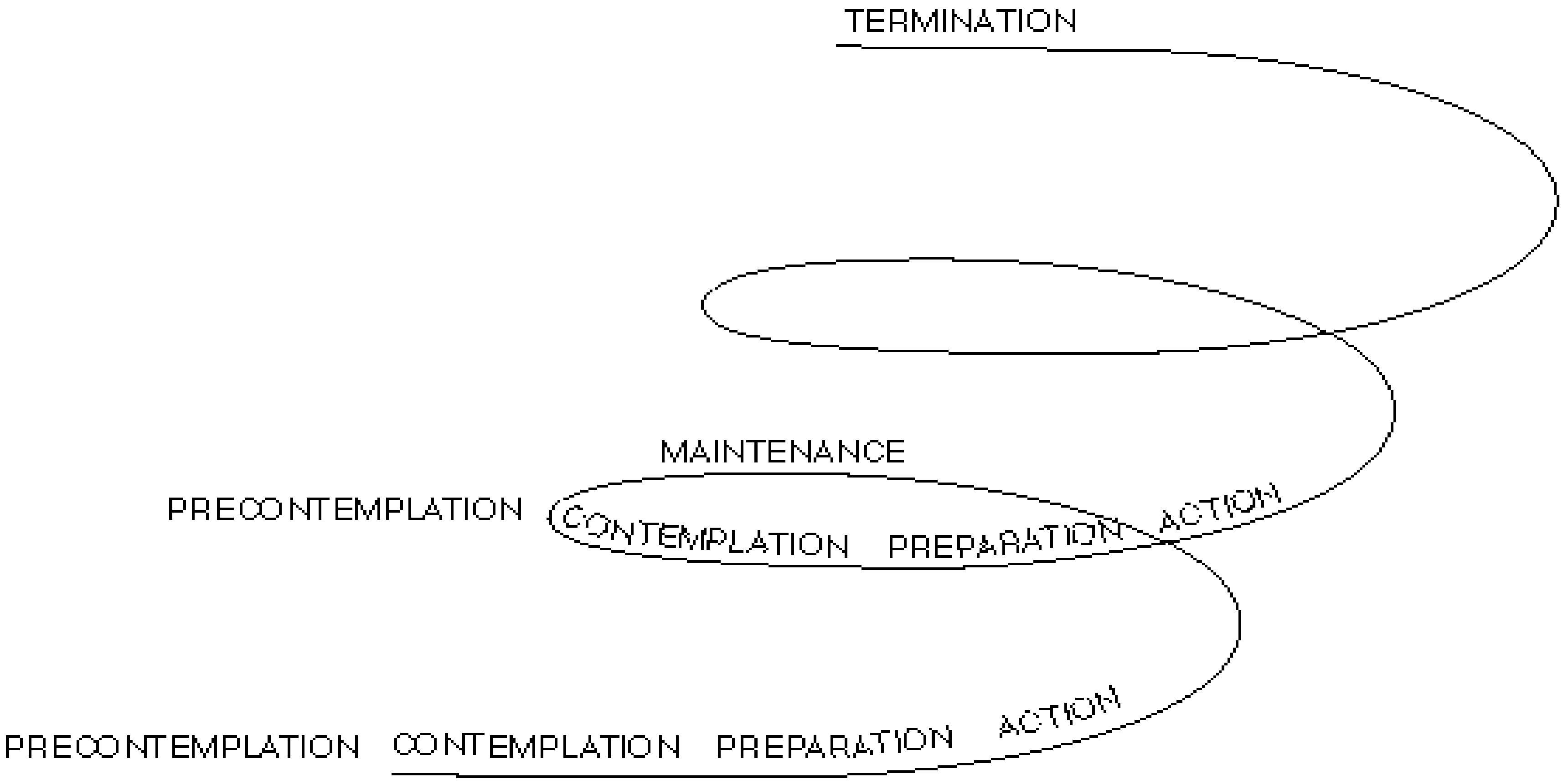
MATH ACADEMY
UPPER ELEMENTARY

Robyn Seifert Decker
UltraMathPD@gmail.com

AGENDA

- Place Value
- Addition
- Subtraction
- Problem Solving
- Fractions
- If time allows: Multiplication and Division


Change



Spiral of change

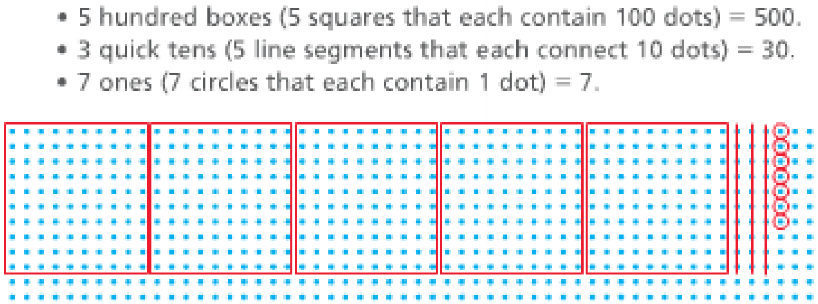
From Prochaska, DiClemente & Norcross, 1992, p1104

MATH PRACTICES

- A teacher every day asks:
- Did I do math sense-making about math structure using math drawings to support math explaining?
- Can I do some part of this better tomorrow?

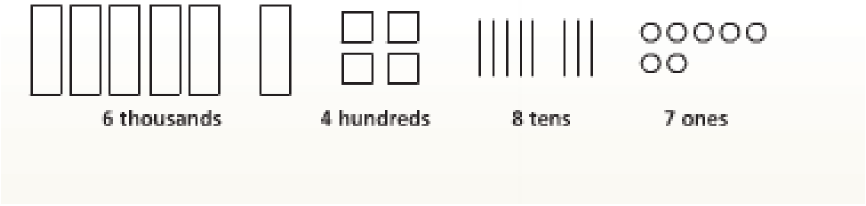
BASE-TEN UNITS

- Place value drawings to help conceptualize numbers and understand the relative size of place values



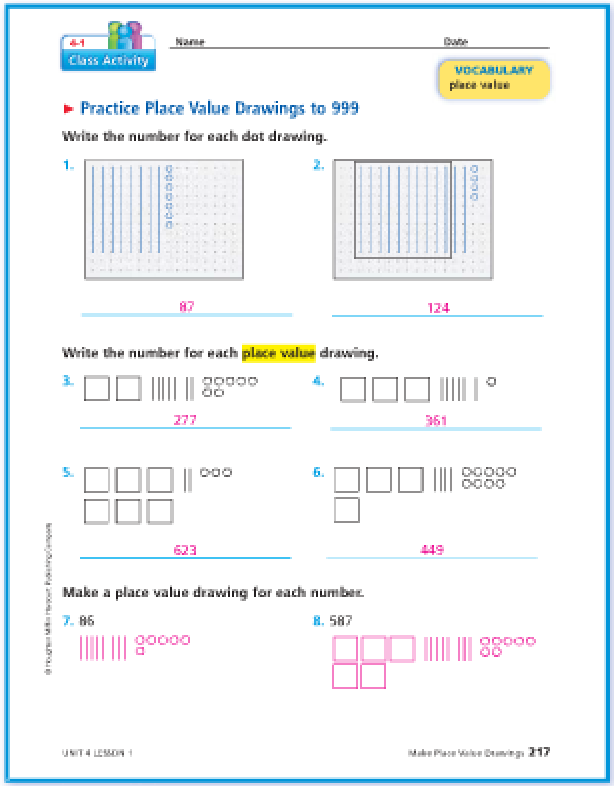
- 5 hundred boxes (5 squares that each contain 100 dots) = 500.
- 3 quick tens (5 line segments that each connect 10 dots) = 30.
- 7 ones (7 circles that each contain 1 dot) = 7.

- Once concept of the 1s is understood move to drawings without dots



PLACE VALUE DRAWINGS

- Reason Abstractly and Quantitatively
  - How much is a box worth?
  - How much is a stick worth?
  - How much is a circle worth?
  - How can you count to find the number shown in the drawing?
- Students should be able to explain the “10 times” pattern for ones, tens, hundreds, and thousands.
  - Understanding that each place value is 10 times as great as the value of the previous place.



4-1 Class Activity

Name Date

VOCABULARY
place value

Practice Place Value Drawings to 999

Write the number for each dot drawing.

1. 87

2. 124

Write the number for each place value drawing.

3. 277

4. 361

5. 623

6. 449

Make a place value drawing for each number.

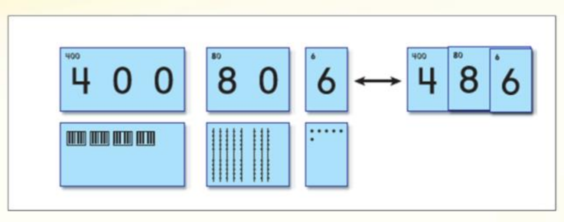
7. 86

8. 587

UNIT 4 LESSON 1

Make Place Value Drawings 217

SECRET CODE CARDS



Tools

- Grade 3 the quantities are on the back
  - Expanded form gives the English repeating counting sequence
  - Standard form you read the English number and the position tells the value
  - Word form shows the values in words
    - Our number system is a base ten within a base thousand system
      - So we say 468thousand, 2hundred35

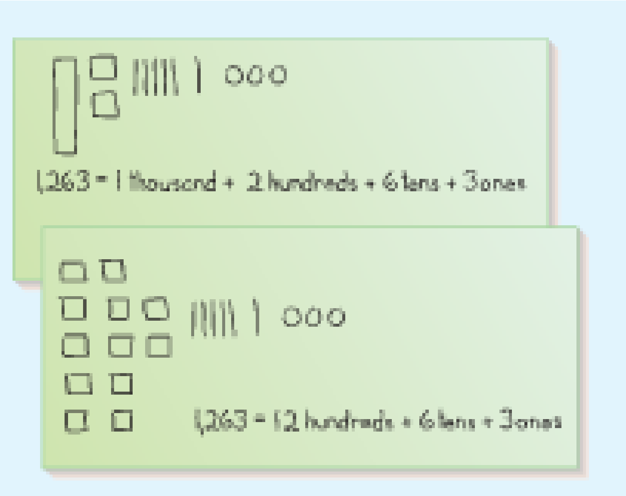
Structure

- Value of the digit 1 differs in each place
- Value to the left is multiplied by 10
- Value to the right is divided by 10

MATH TALK

Using secret code cards as a tool to provide structure…

- The importance of the place value to explain how to find the expanded form of the number 1,263.



Digit x Place Value = Total Value

1 x 1,000 = 1,000

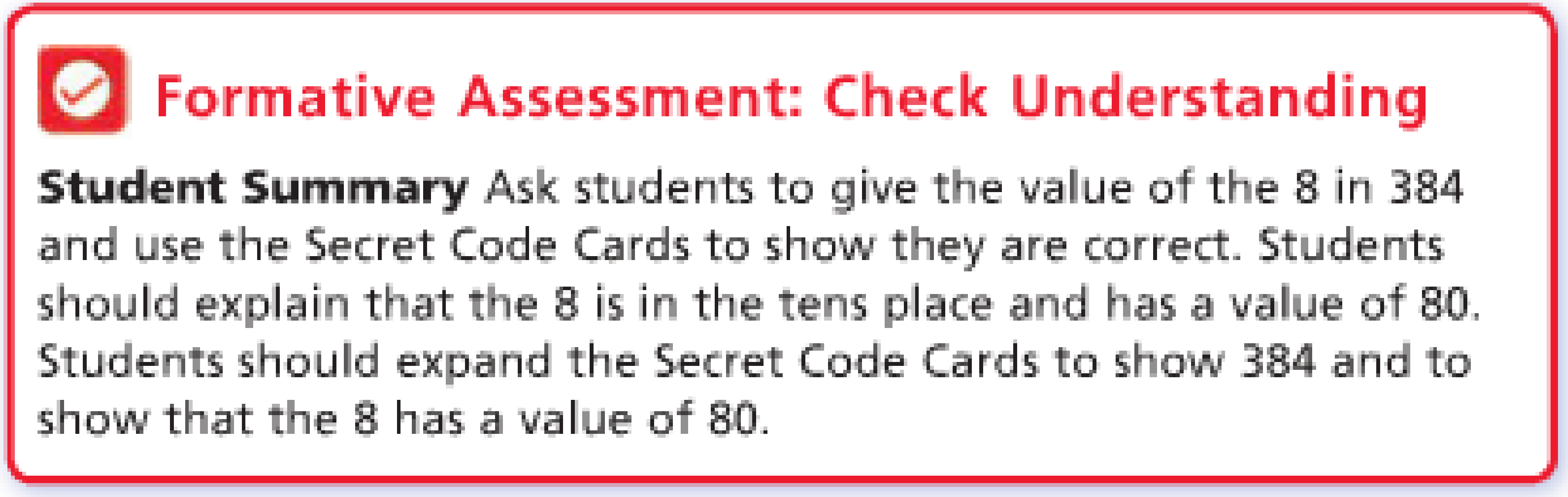
1,000 is the total value of 1

2 x 100 = 200, 6 x 10 = 60, 3 x 1 = 3

Combine total values 1,000 + 200 + 60 + 3

What's another way to read this number?

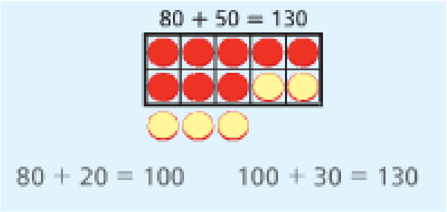
DISCUSS WITH YOUR TABLE



Formative Assessment: Check Understanding

Student Summary Ask students to give the value of the 8 in 384 and use the Secret Code Cards to show they are correct. Students should explain that the 8 is in the tens place and has a value of 80. Students should expand the Secret Code Cards to show 384 and to show that the 8 has a value of 80.

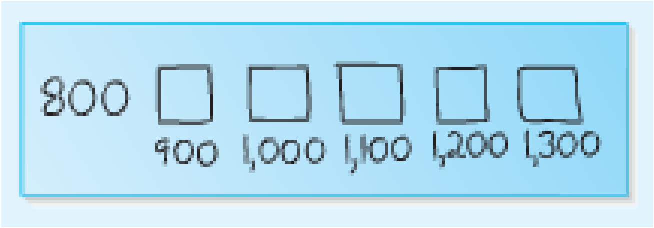
- Understanding how numbers can be grouped and ungrouped in different ways is important to developing and understanding methods for multi-digit addition and subtraction.



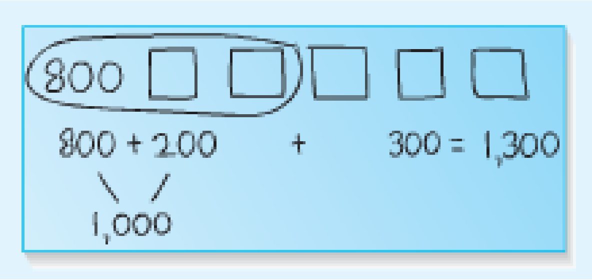
MAKE A TEN, HUNDRED, THOUSAND STRATEGY

- Counting-On by Tens
  - 80 + 50
    - Think 8 tens + 5 tens
      - Say: 8 tens
      - Count on until you have counted 5 tens: 9 tens, 10 tens, 11 tens, 12 tens, 13 tens. The answer is 13 tens or 130

- Counting-On by Hundreds

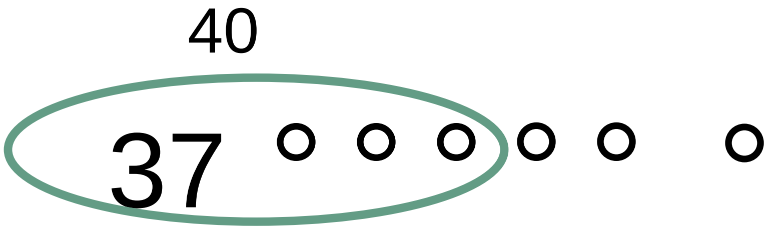


- Counting-On by Thousands



GROUP A NEW DECADE

- Draw a proof picture using the “Count-on” strategy to find the answer to 37 + 6.



40

37

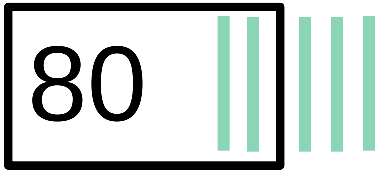
40 + 3 = 43

- Circle the next decade number in the drawing.
- Label the decade number in the drawing.
- Write the “decade” equation.
- Now try and discuss: **48 + 5 and 76 + 8.**

GROUP A NEW TEN, HUNDRED, THOUSAND

- Draw a proof picture using the “Count-on” strategy to find the answer to 80 + 50.

80 + 20 = 100



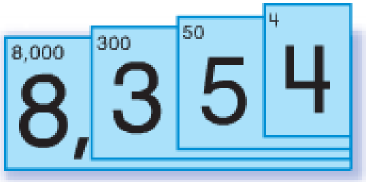
100 + 30 = 130

80 + 50

- Draw the hundred number in the drawing.
- Label the hundred number in the drawing.
- Write the “hundred” equation.
- Now try and discuss: 700 + 500 and 900 + 600.

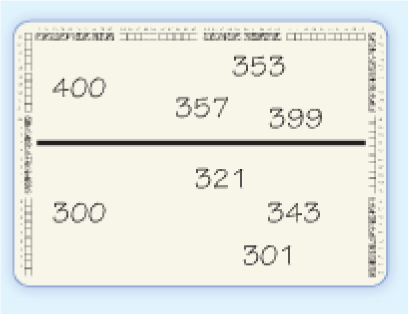
ROUNDING

- Tools
  - Discuss the value and position of each digit



- Precision
  - Explain when rounding we look to the right of the hundreds place and because it is 5, we round 354 up to 400

- Abstractly and Quantitatively



The Meaningful Development of Standard Algorithms in the CCSS-M

The CCSS-M conceptual approach to computation is deeply mathematical and enables students to make sense of and use the base ten system and properties of operations powerfully. The CCSS-M focus on understanding and explaining such calculations, with the support of visual models, enables students to see mathematical structure as accessible, important, interesting, and useful.

The relationships across operations are also a critically important mathematical idea. How the regularity of the mathematical structure in the base ten system can be used for so many different kinds of calculation is an important feature of what we want students to appreciate in the elementary grades.

It is crucial to use the Standards of Mathematical Practice throughout the development of computational methods.

Misconceptions about the CCSS-M and the NBT Progression

These are all wrong.

The standard algorithm is the method I learned.

The standard algorithm is the method commonly taught now (the current common method).

There is only one way to write the algorithm for each operation.

The standard algorithm means teaching by rote without understanding.

Teachers or programs may not teach the standard algorithm until the grade at which fluency is specified in the CCSS-M.

Initially teachers or programs may only use methods that children invent.

Teachers or programs must emphasize special strategies useful only for certain numbers.

What Is the Standard Algorithm?

The NBT Progression document summarizes that *the standard algorithm* for an operation implements the following mathematical approach

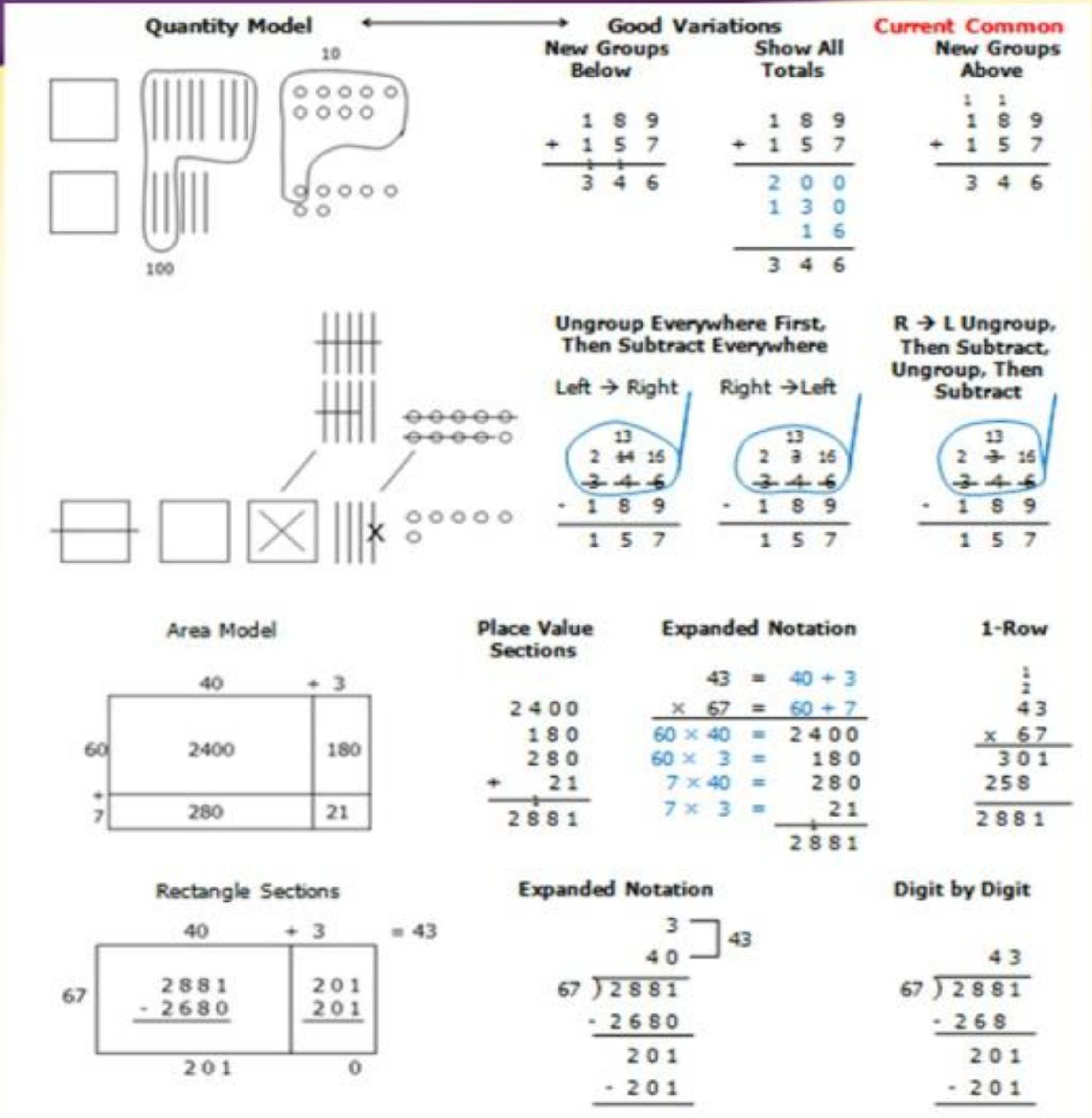
with minor variations in how the algorithm is written:

- decompose numbers into base-ten units and then carry out single-digit computations with those units using the place values to direct the place value of the resulting number; and
- use the one-to-ten uniformity of the base ten structure of the number system to generalize to large whole numbers and to decimals.

To implement a standard algorithm one uses a systematic written method for recording the steps of the algorithm.

JUST LOOK AT OVERALL STRUCTURE

Drawings and Written Variations of Standard Algorithms



Quantity Model ↔ **Good Variations** **Current Common**

New Groups Below

```
  1 8 9
+ 1 5 7
-------
  3 4 6
```

Show All Totals

```
  1 8 9
+ 1 5 7
-------
  2 0 0
  1 3 0
    1 6
-------
  3 4 6
```

New Groups Above

```
  1 1
  1 8 9
+ 1 5 7
-------
  3 4 6
```

Ungroup Everywhere First, Then Subtract Everywhere

Left → Right

```
     13
  2 14 16
  ~~3 4 6~~
- 1  8  9
---------
  1  5  7
```

Right → Left

```
     13
  2  3 16
  ~~3 4 6~~
- 1  8  9
---------
  1  5  7
```

R → L Ungroup, Then Subtract, Ungroup, Then Subtract

```
     13
  2  3 16
  ~~3 4 6~~
- 1  8  9
---------
  1  5  7
```

Area Model

Place Value Sections

```
  2400
   180
   280
+   21
------
  2881
```

Expanded Notation

```
       43  =  40 + 3
   ×   67  =  60 + 7
60 × 40    =  2400
60 ×  3    =   180
 7 × 40    =   280
 7 ×  3    =    21
              ----
              2881
```

1-Row

```
   1
   2
   43
 × 67
 ----
  301
 258
 ----
 2881
```

Rectangle Sections

Expanded Notation

```
        3 ]
       40 ] 43
67 ) 2881
    -2680
      201
     -201
```

Digit by Digit

```
       43
67 ) 2881
    -268
      201
     -201
```

EXPLORE ADDITION METHODS

- Proof Drawings support the development of place value language.
  - Expanded Notation
  - Show All Totals (Left to Right)
  - Show All Totals (Right to Left)
  - New Ten Groups Below
  - New Ten Groups Above

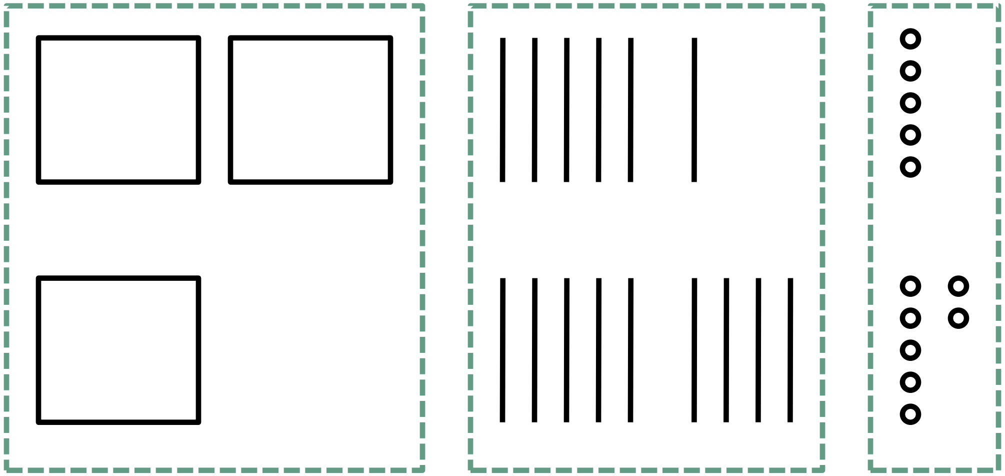
EXPANDED NOTATION

Step 1: *Expand each number*

Step 2: *Add the hundreds*

Step 3: *Add the tens*

Step 4: *Add the ones*



Step 5: *Rewrite in standard form*

$$\begin{array}{rcl} 265 & = & 200 + 60 + 5 \\ +\,197 & = & 100 + 90 + 7 \\ \hline \mathbf{462} & = & 300 + 150 + 12 \end{array}$$

SHOW ALL TOTALS (LEFT TO RIGHT)

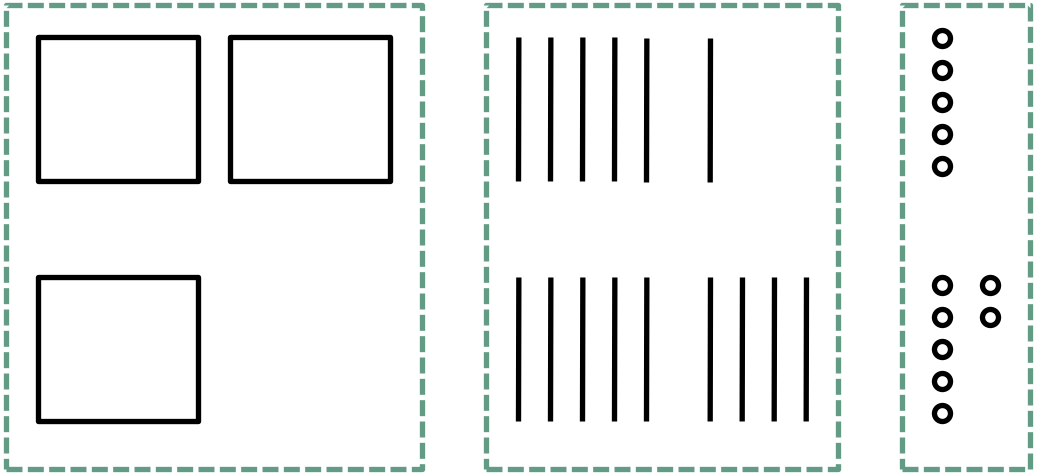
Step 1: *Add the hundreds*

Step 2: *Add the tens*

Step 3: *Add the ones*

Step 4: *Add the Sub-totals*

$$\begin{array}{r} 2\ 6\ 5 \\ +\ 1\ 9\ 7 \\ \hline 3\ 0\ 0 \\ 1\ 5\ 0 \\ +\ \ 1\ 2 \\ \hline \mathbf{4\ 6\ 2} \end{array}$$



SHOW ALL TOTALS (RIGHT TO LEFT)

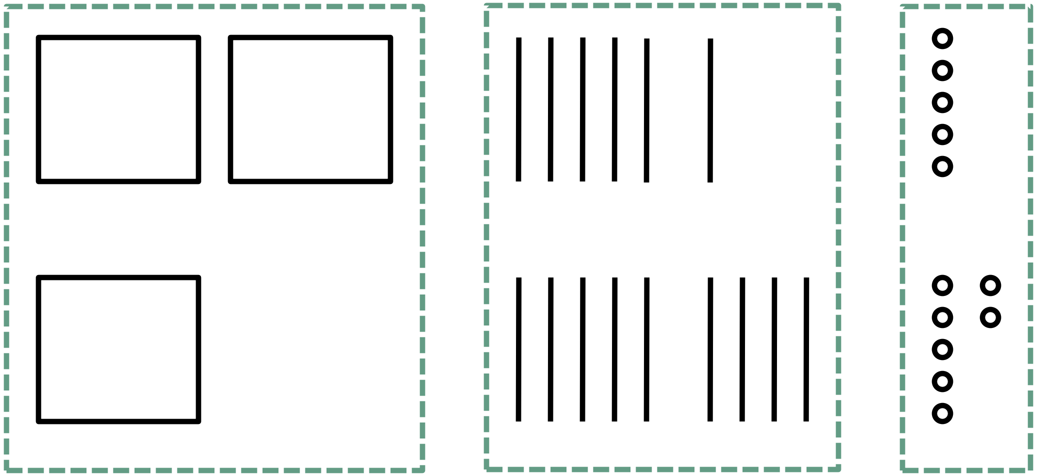
***Step 1:** Add the ones*

***Step 2:** Add the tens*

***Step 3:** Add the hundreds*

***Step 4:** Add the Sub-totals*

2 6 5
\+ 1 9 7
1 2
1 5 0
\+ 3 0 0
4 6 2



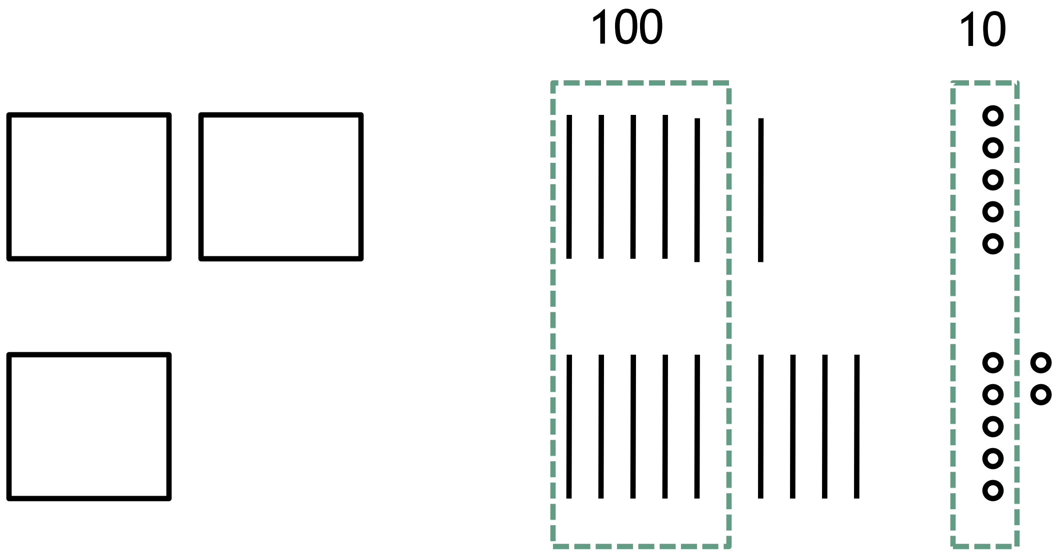
NEW GROUPS BELOW

Step 1: *Add the ones*
(Show the new ten if possible)

Step 2: *Add the tens*
(Show the new hundred if possible)

Step 3: *Add the hundreds*

$$\begin{array}{rccc} & 2 & 6 & 5 \\ + & 1 & 9 & 7 \\ & 1 & 1 & \\ \hline & 4 & 6 & 2 \end{array}$$



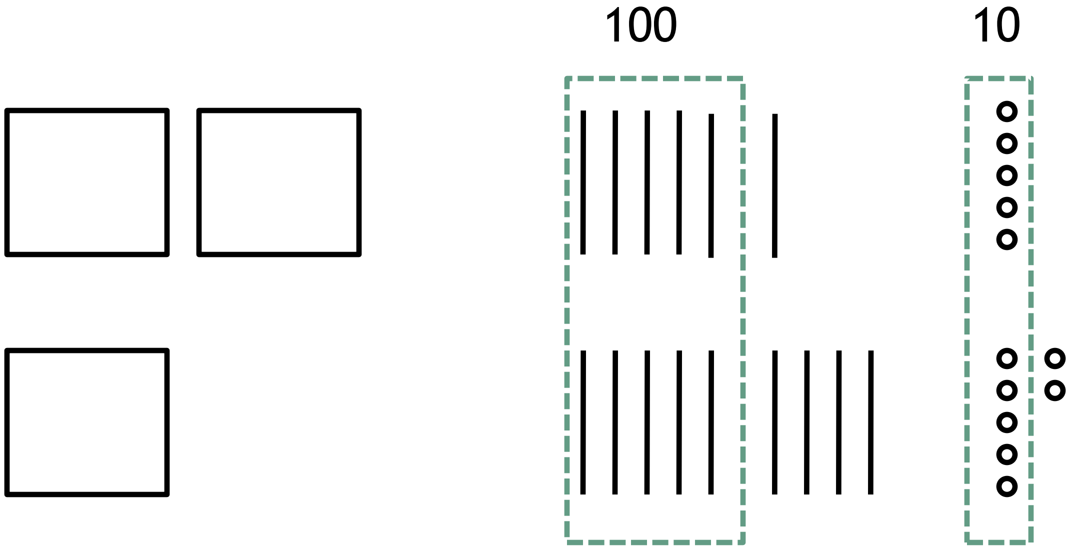
NEW GROUPS ABOVE

Step 1: *Add the ones*
(Show the new ten if possible)

Step 2: *Add the tens*
(Show the new hundred if possible)

Step 3: *Add the hundreds*

$$\begin{array}{rccc} & 1 & 1 & \\ & 2 & 6 & 5 \\ + & 1 & 9 & 7 \\ \hline & 4 & 6 & 2 \end{array}$$



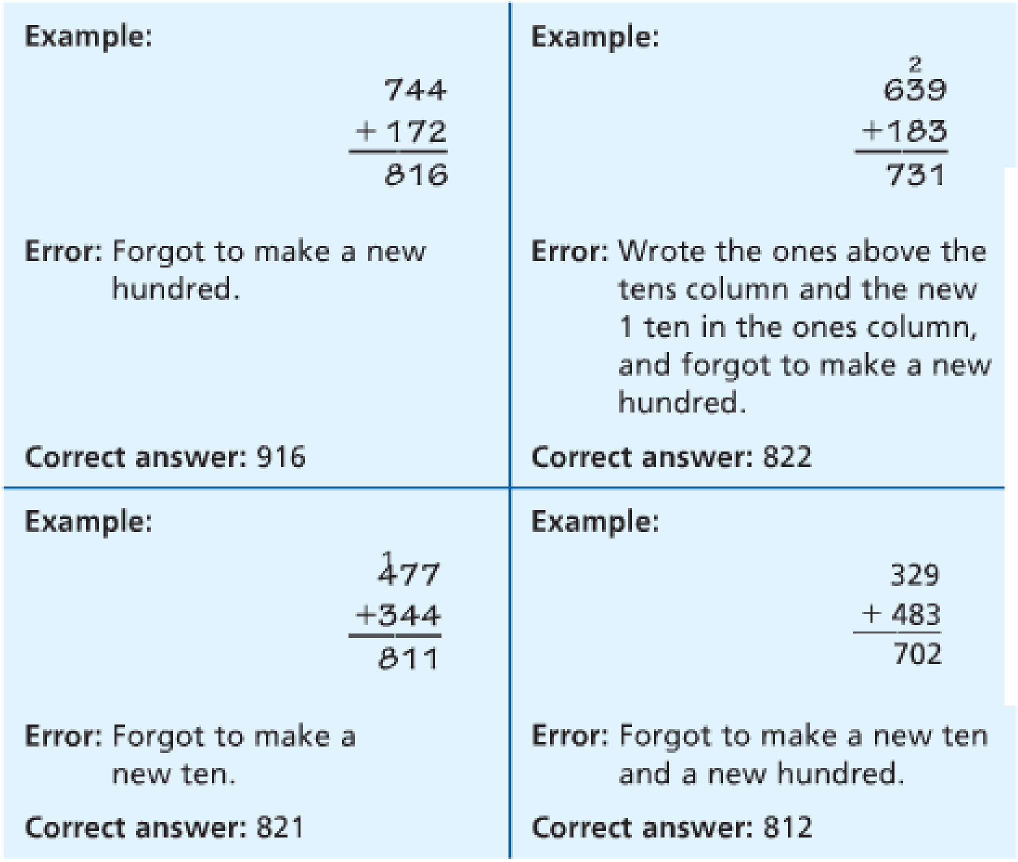
INDEPENDENT PRACTICE - ADDITION METHODS

- Please turn to the “Independent Practice” and do on your own.

- When finished, compare and discuss your work as a table group.

HELP THE TEACHER

- Have students suggest strategies to avoid the common errors you make for an activity.
- Discuss at your table how this lesson may look in your classroom.



Example:

$$\begin{array}{r} 744 \\ +172 \\ \hline 816 \end{array}$$

Error: Forgot to make a new hundred.

Correct answer: 916

Example:

$$\begin{array}{r} \overset{2}{6}39 \\ +183 \\ \hline 731 \end{array}$$

Error: Wrote the ones above the tens column and the new 1 ten in the ones column, and forgot to make a new hundred.

Correct answer: 822

Example:

$$\begin{array}{r} \overset{1}{4}77 \\ +344 \\ \hline 811 \end{array}$$

Error: Forgot to make a new ten.

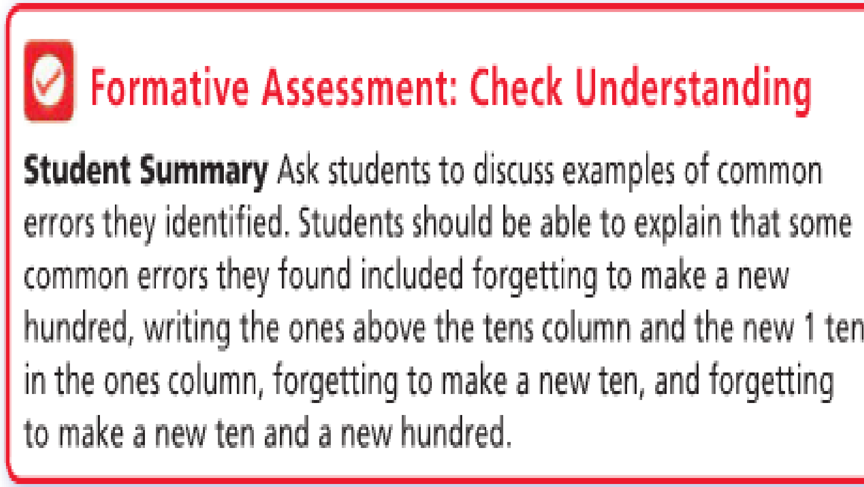
Correct answer: 821

Example:

$$\begin{array}{r} 329 \\ +483 \\ \hline 702 \end{array}$$

Error: Forgot to make a new ten and a new hundred.

Correct answer: 812

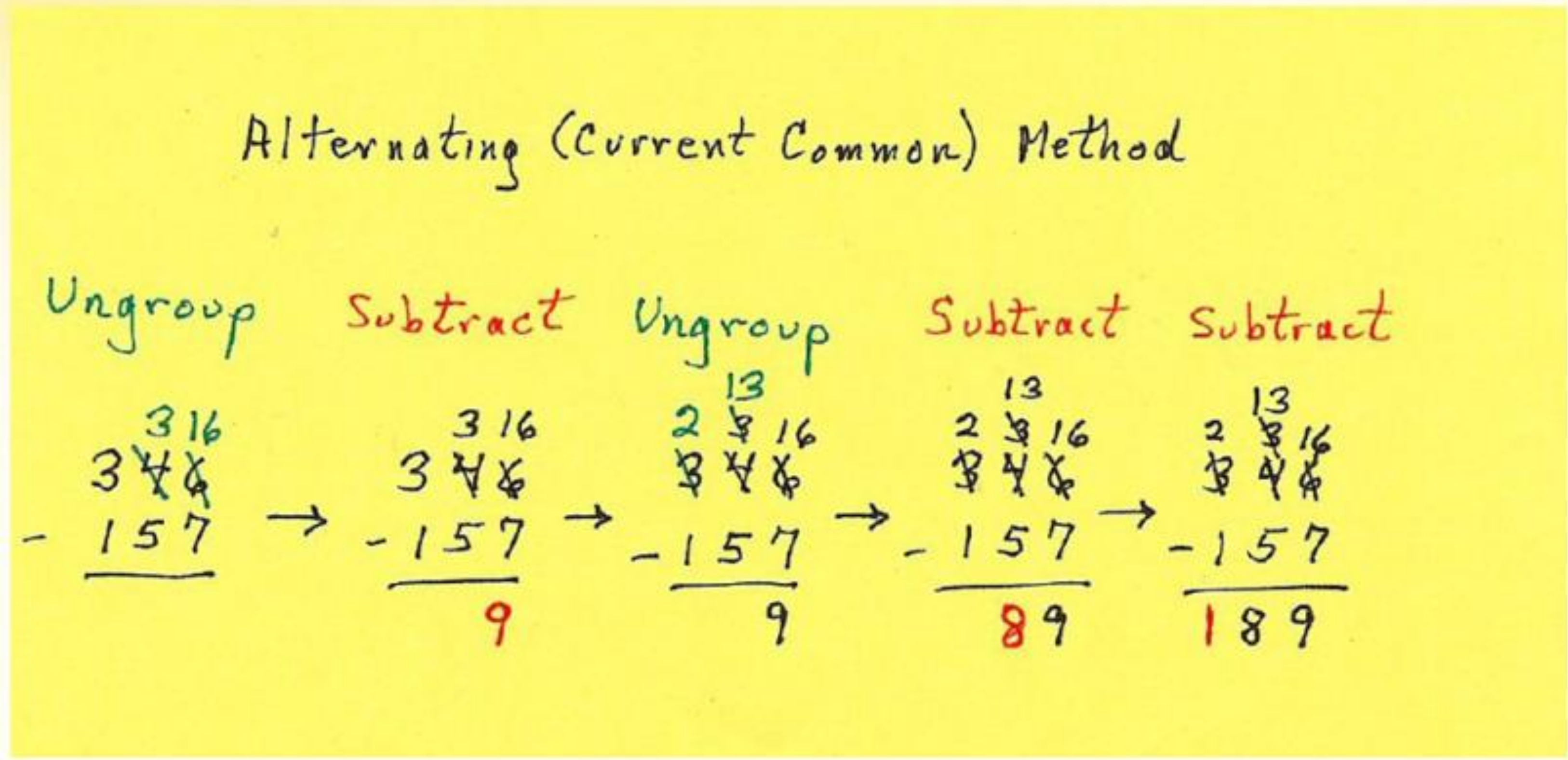


Formative Assessment: Check Understanding

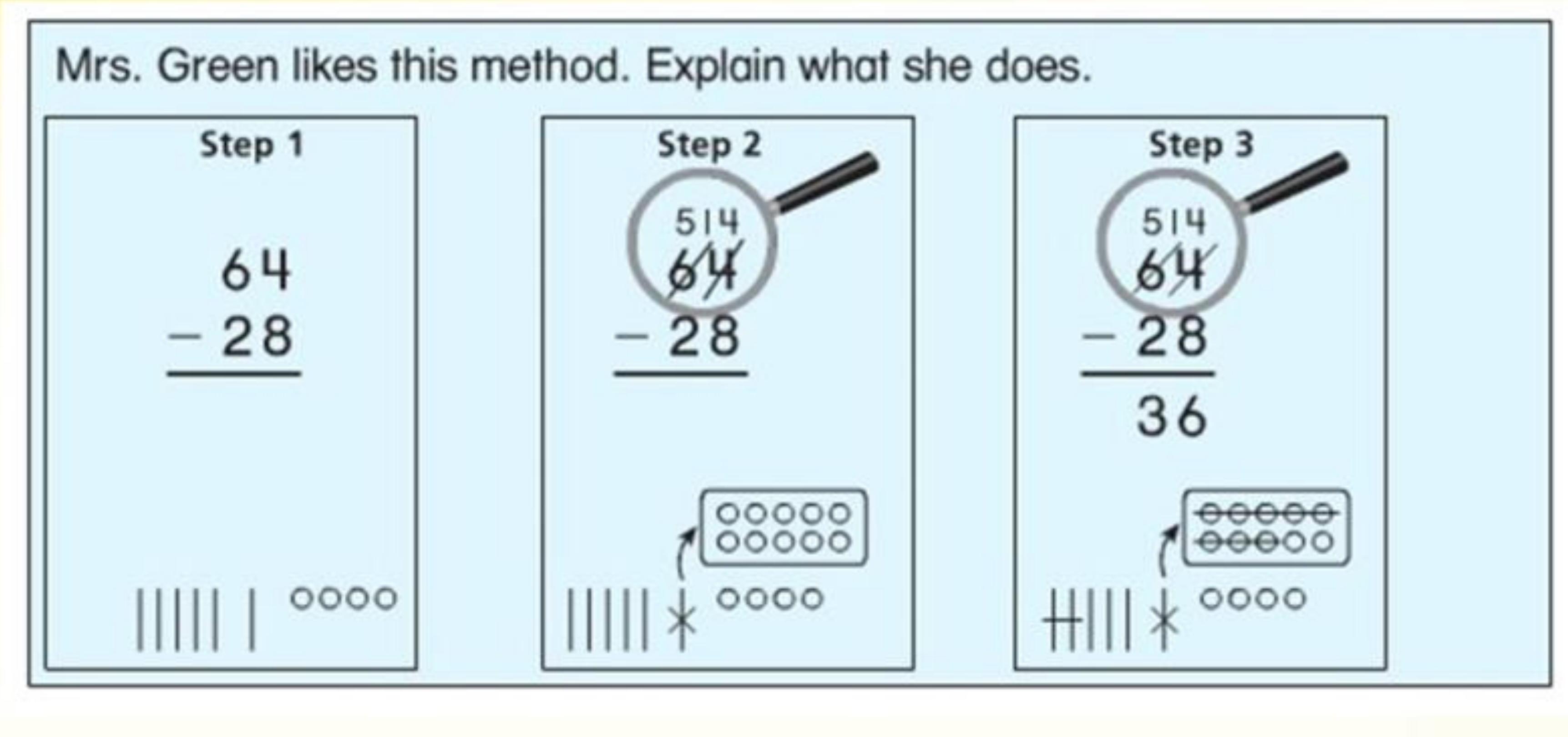
Student Summary Ask students to discuss examples of common errors they identified. Students should be able to explain that some common errors they found included forgetting to make a new hundred, writing the ones above the tens column and the new 1 ten in the ones column, forgetting to make a new ten, and forgetting to make a new ten and a new hundred.

COMMON SUBTRACTION METHOD

- Do you see a potential for errors?



The Ungroup First Method Within 100



Mrs. Green likes this method. Explain what she does.

- What are potential benefits to ungrouping first?

EXPLORE SUBTRACTION METHODS

- Reminder: Proof Drawings support the development of place value language.
  - Expanded Notation
  - Ungroup First (Right to Left)
  - Ungroup First (Left to Right)

EXPANDED NOTATION

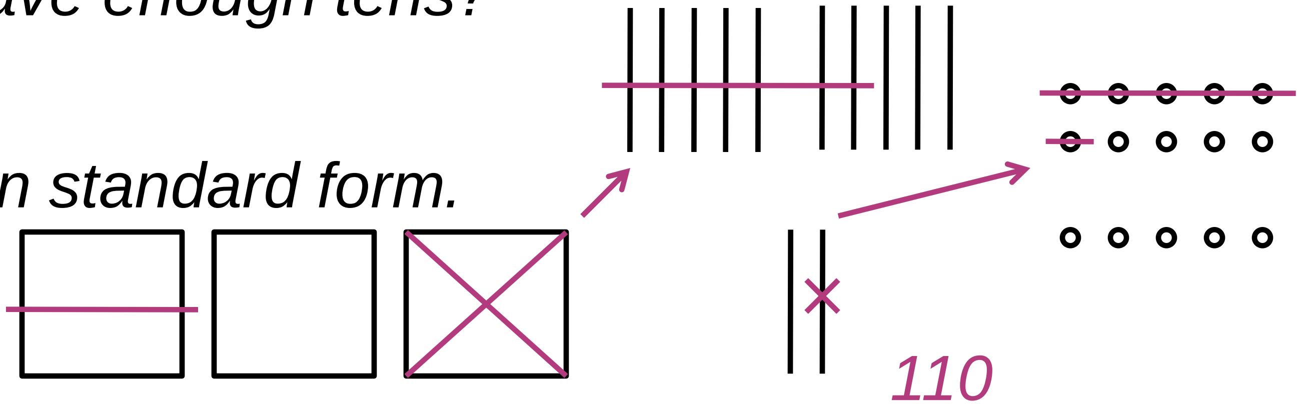
Step 1: *Draw 325 and expand both numbers.*

Step 2: *Do we have enough ones?*

Step 3: *Do we have enough tens?*

Step 4: *Subtract*

Step 5: *Rewrite in standard form.*



$$\begin{array}{rcrcrcr} 325 & = & \overset{200}{\cancel{300}} & + & \overset{\overset{110}{\cancel{10}}}{\cancel{20}} & + & \overset{15}{\cancel{5}} \\ -\ 176 & = & -\ 100 & & -\ 70 & & -\ 6 \\ \hline 149 & = & 100 & & +\ 40 & & +\ 9 \end{array}$$

UNGROUP FIRST (RIGHT TO LEFT)

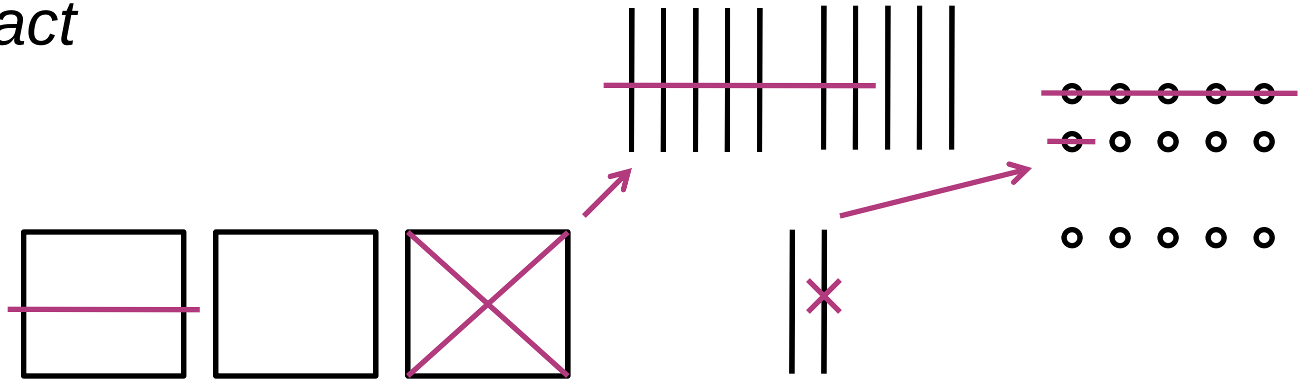
Step 1: *Draw 325.*

Step 2: *Do we have enough ones?*

Step 3: *Do we have enough tens?*

Step 4: *Subtract*

$$\begin{array}{r} \overset{2}{\cancel{3}}\ \overset{\overset{11}{\cancel{1}}}{\cancel{2}}\ \overset{15}{\cancel{5}} \\ -\ 1\ 7\ 6 \\ \hline 1\ 4\ 9 \end{array}$$



UNGROUP FIRST (LEFT TO RIGHT)

Step 1: *Draw 325.*

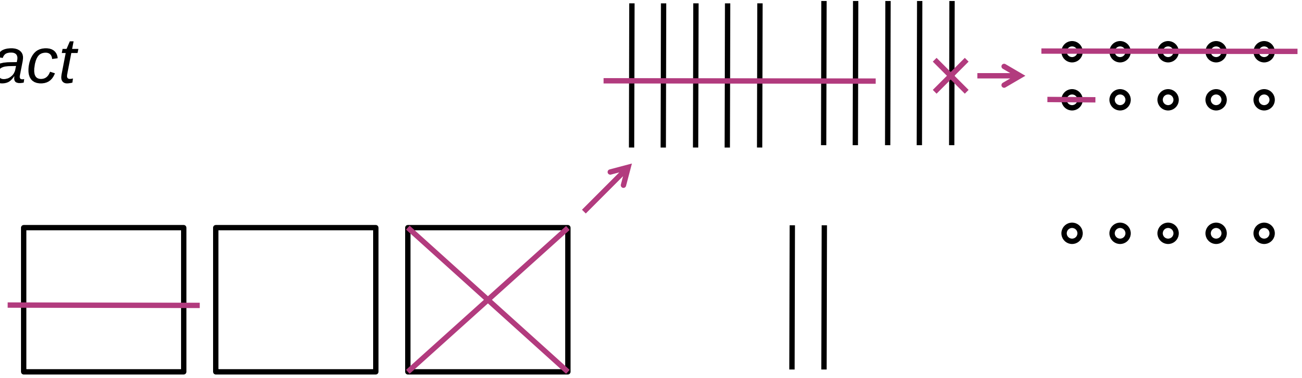
Step 2: *Do we have enough hundreds?*

Step 3: *Do we have enough tens?*

Step 4: *Do we have enough ones?*

Step 5: *Subtract*

$$\begin{array}{r} 2\ \overset{11}{\cancel{12}}\ 15 \\ \cancel{3}\ \cancel{2}\ \cancel{5} \\ -\ 1\ 7\ 6 \\ \hline 1\ 4\ 9 \end{array}$$



UNGROUP FIRST (LEFT TO RIGHT) WITH ZEROS

Step 1: *Draw 105.*

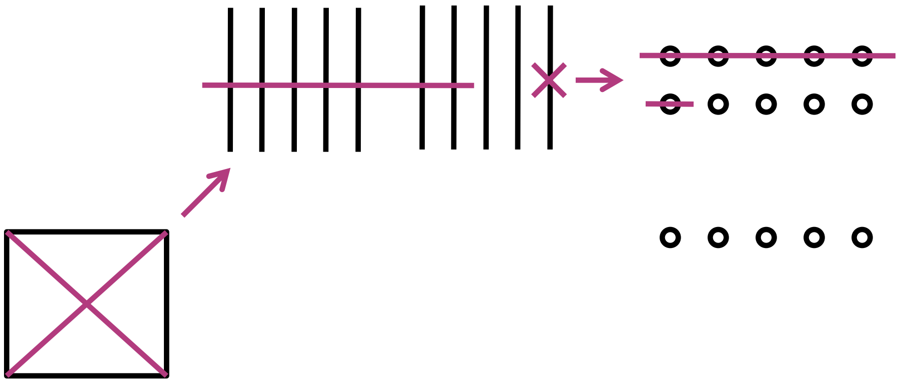
Step 2: *Do we have enough hundreds?*

Step 3: *Do we have enough tens?*

Step 4: *Do we have enough ones?*

Step 5: *Subtract*

$$\begin{array}{r} \overset{0}{\cancel{1}}\;\overset{\overset{9}{\cancel{10}}}{\cancel{0}}\;\overset{15}{\cancel{5}} \\ -\quad 7\;\;6 \\ \hline 2\;\;9 \end{array}$$



INDEPENDENT PRACTICE - SUBTRACTION METHODS

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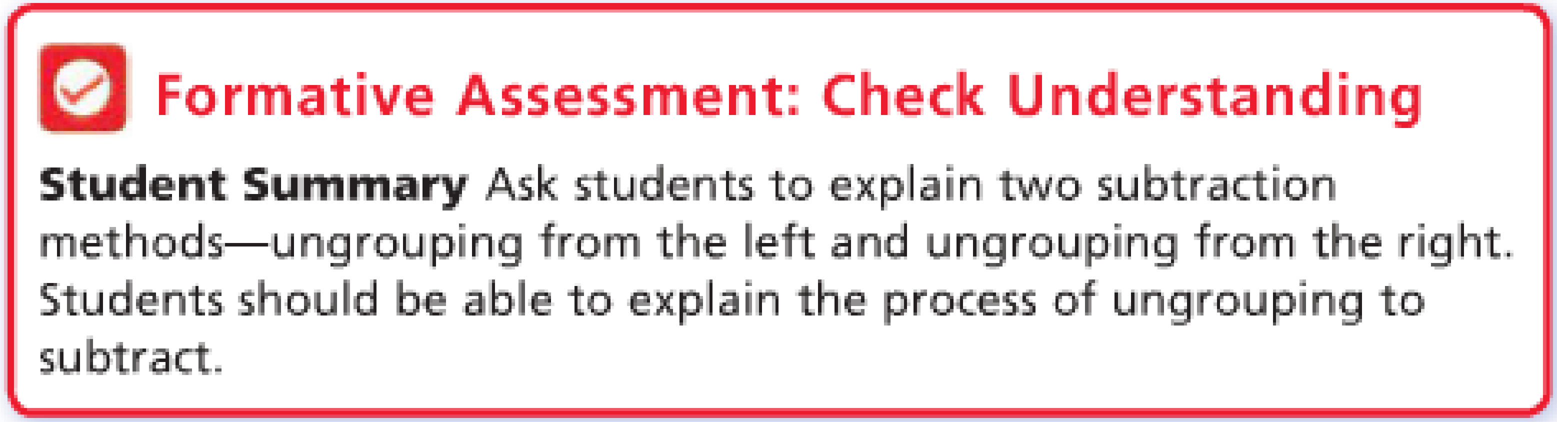
- When finished, compare and discuss your work as a table group.



Formative Assessment: Check Understanding

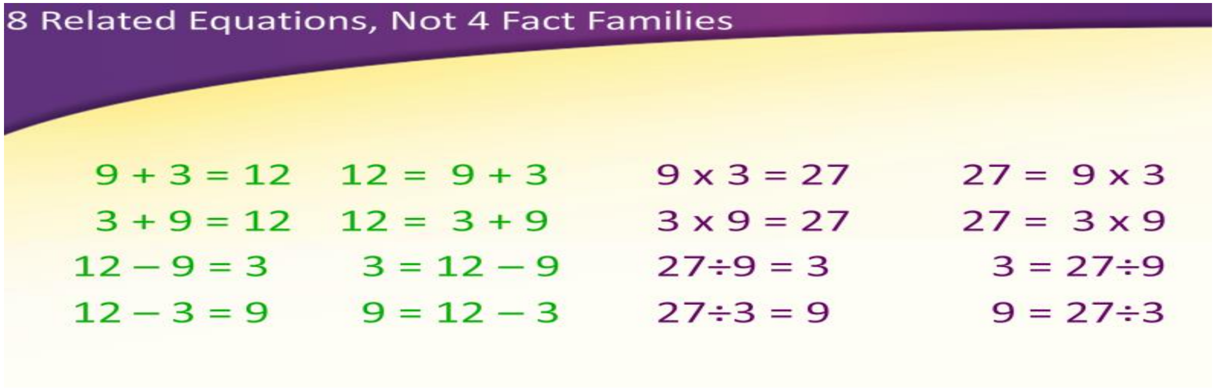
Student Summary Ask students to explain two subtraction methods—ungrouping from the left and ungrouping from the right. Students should be able to explain the process of ungrouping to subtract.

PROBLEM SOLVING PROCESS

- Understand the situation
  - Make sense of the language – to conceptualize the real world situation
    - Make sense of the problem
    - Reason Abstractly and quantitatively
- Represent the situation with a drawing/situation equation
  - Mathematize the situation – focus on mathematical aspects of situation
    - Model with mathematics
    - Look for and make use of structure
- Solve the representation (write a solution equation)
  - Find the answer – use drawings/situation/solution equation
    - Use appropriate tools
    - Use repeated reasoning
- Check the answer makes sense
  - Check the answer in the context of the problem – write and explain the label and answer
    - Critique the reasoning of others
    - Attend to precision

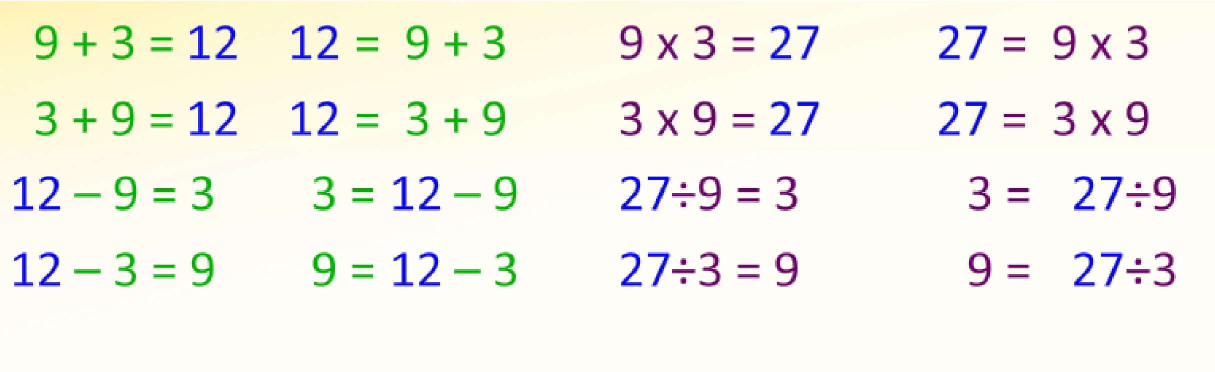
RELATING EQUATIONS

- Becoming flexible problem solvers
- Understanding the product on either side of the equation



8 Related Equations, Not 4 Fact Families

9 + 3 = 12	12 = 9 + 3	9 x 3 = 27	27 = 9 x 3
3 + 9 = 12	12 = 3 + 9	3 x 9 = 27	27 = 3 x 9
12 – 9 = 3	3 = 12 – 9	27÷9 = 3	3 = 27÷9
12 – 3 = 9	9 = 12 – 3	27÷3 = 9	9 = 27÷3

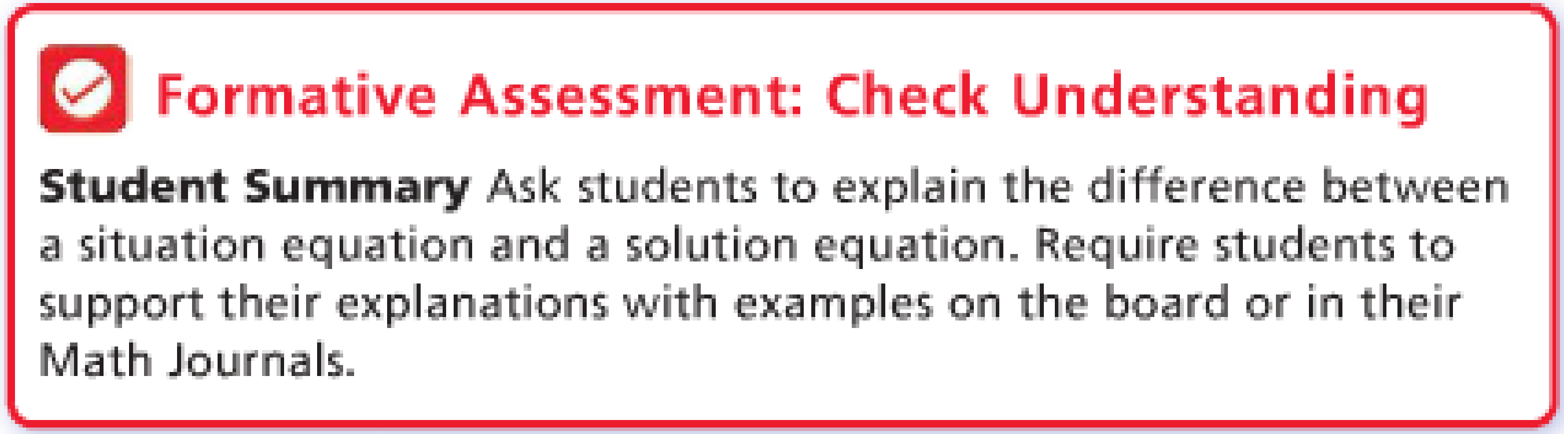


9 + 3 = 12	12 = 9 + 3	9 x 3 = 27	27 = 9 x 3
3 + 9 = 12	12 = 3 + 9	3 x 9 = 27	27 = 3 x 9
12 – 9 = 3	3 = 12 – 9	27÷9 = 3	3 = 27÷9
12 – 3 = 9	9 = 12 – 3	27÷3 = 9	9 = 27÷3

REPRESENTING THE SITUATION

Operations and Algebraic thinking

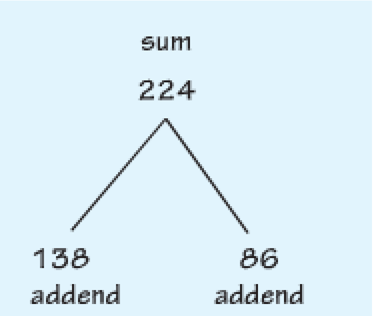
- Represent the situation with a drawing, diagram and/or equation
  - A **situation** equation shows the action or the relationships in a problem
- Then decide how to solve for the answer
  - A **solution** equation shows the operation that is performed to solve the problem



Formative Assessment: Check Understanding

Student Summary Ask students to explain the difference between a situation equation and a solution equation. Require students to support their explanations with examples on the board or in their Math Journals.

MAKE SENSE OF PROBLEMS



- Connect diagrams and equations
  - Solve problem 2 without doing any work

1. There were 138 students in the gym for the assembly. Then 86 more students came in. How many students were in the gym altogether?

2. There were 224 students in the gym for the assembly. Then 86 students left. How many students were still in the gym?

- 138 + 86 = 224
  addend + addend = sum
- 224 – 86 = 138
  sum - addend = addend

- Addition and subtraction undo each other

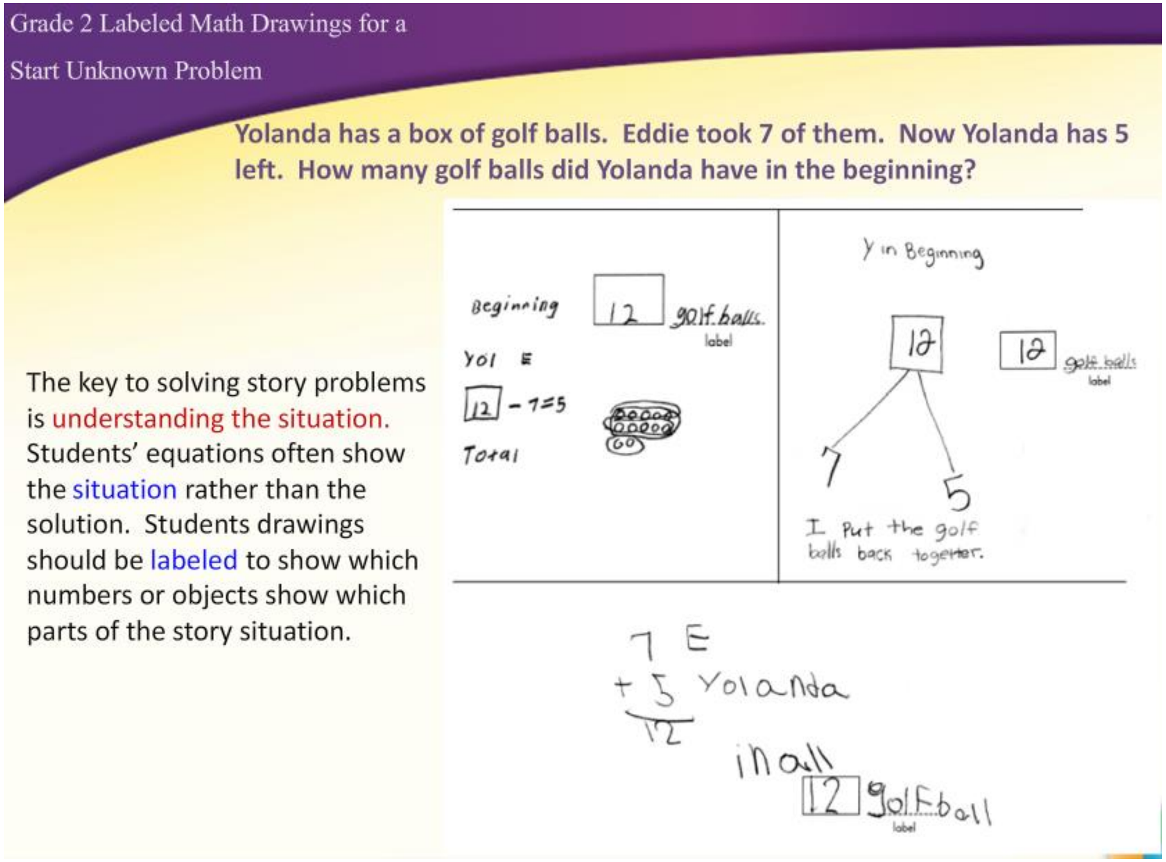
ONE STEP WORD PROBLEM

Shayna had some markers. She gave 5 of the markers to her friends. Now she has 2 markers. How many markers did she have in the beginning?

- How many markers did Shayna give away?
  - 5
- How many markers did Shayna have left?
  - 2
- What are you trying to find out?
  - The number of markers Shayna had when she started
- Is this the unknown number in the situation?
  - Yes
- How can we find the unknown number or solution?
  - Add 5+2 (the number of markers that Shayna gave away plus the number of markers that were left)
- Situation: $b - 5 = 2$ Solution: $b = 2 + 5$

LABEL MATH DRAWINGS

- Key is understanding the situation
- Labeling explains the parts of the story



Grade 2 Labeled Math Drawings for a Start Unknown Problem

Yolanda has a box of golf balls. Eddie took 7 of them. Now Yolanda has 5 left. How many golf balls did Yolanda have in the beginning?

The key to solving story problems is understanding the situation. Students' equations often show the situation rather than the solution. Students drawings should be labeled to show which numbers or objects show which parts of the story situation.

I Put the golf balls back togetter.

ADDITION COMPARISON

- Look at the problem below.
  - The word more might lead students to believe that they should add, but they actually need to subtract
  - They know the greater quantity and the difference, so subtraction will give them the lesser quantity
- What took more time? How much more time?
- How do we show this?
- How do comparison bars help?

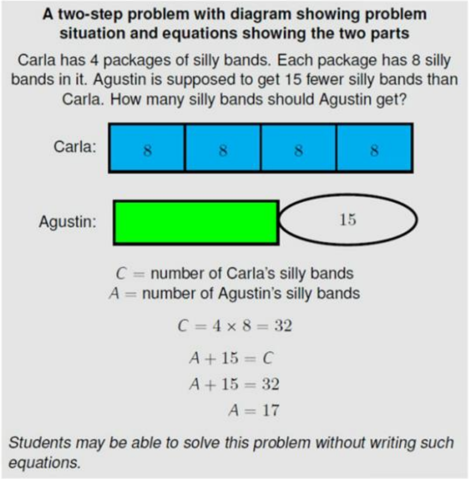
practiced
The soccer team drilled for
practiced
150 minutes last week. The team drilled
for 30 minutes more than it scrimmaged.
game
For how long did the team scrimmage?
game

MULTI-STEP PROBLEMS

- Multiple entry points
- More than one operation
- Identify the helping question(s) or the question(s) that needs to be answered before the final solution can be found

VIDEO: WORD PROBLEMS

- Writing the first step equations
- Represent all steps



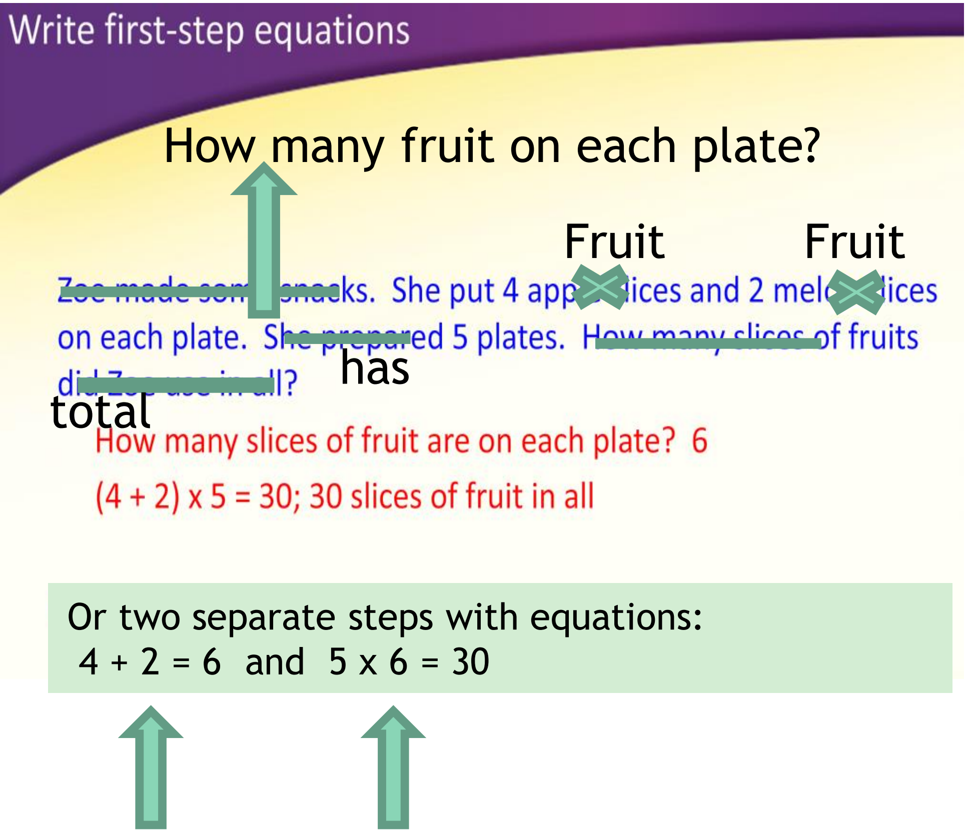
A two-step problem with diagram showing problem situation and equations showing the two parts

Carla has 4 packages of silly bands. Each package has 8 silly bands in it. Agustin is supposed to get 15 fewer silly bands than Carla. How many silly bands should Agustin get?

C = number of Carla's silly bands
A = number of Agustin's silly bands

$$C = 4 \times 8 = 32$$
$$A + 15 = C$$
$$A + 15 = 32$$
$$A = 17$$

Students may be able to solve this problem without writing such equations.



Write first-step equations

How many fruit on each plate?

Fruit Fruit

~~Zoe made some snacks.~~ She put 4 apple slices and 2 melon slices on each plate. S~~he prepared~~ 5 plates. H~~ow many slices of fruits~~ ~~did Zoe use in all~~?

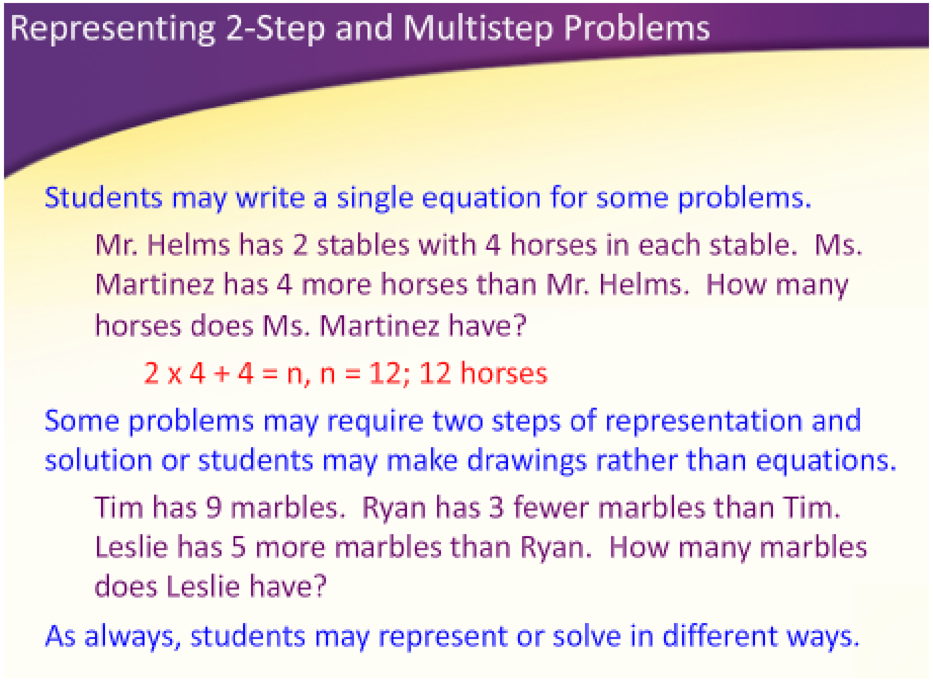
has

total

How many slices of fruit are on each plate? 6

(4 + 2) x 5 = 30; 30 slices of fruit in all

Or two separate steps with equations:
4 + 2 = 6 and 5 x 6 = 30



Representing 2-Step and Multistep Problems

Students may write a single equation for some problems.

Mr. Helms has 2 stables with 4 horses in each stable. Ms. Martinez has 4 more horses than Mr. Helms. How many horses does Ms. Martinez have?

2 x 4 + 4 = n, n = 12; 12 horses

Some problems may require two steps of representation and solution or students may make drawings rather than equations.

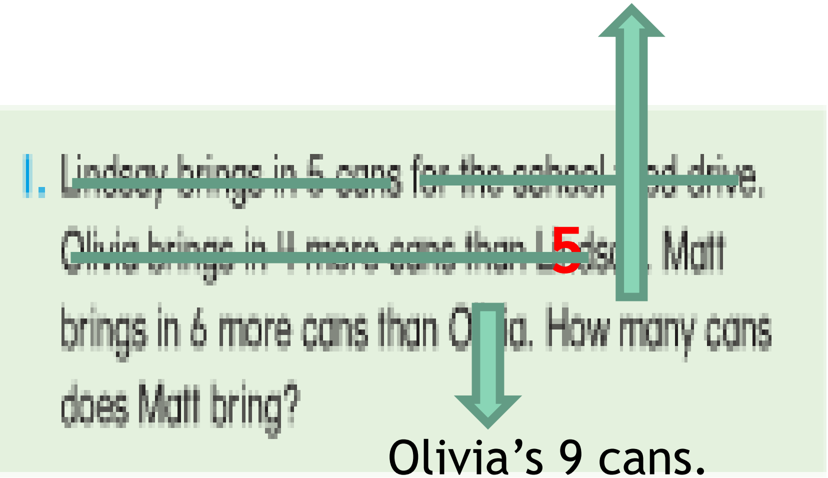
Tim has 9 marbles. Ryan has 3 fewer marbles than Tim. Leslie has 5 more marbles than Ryan. How many marbles does Leslie have?

As always, students may represent or solve in different ways.

TWO STEP WORD PROBLEMS

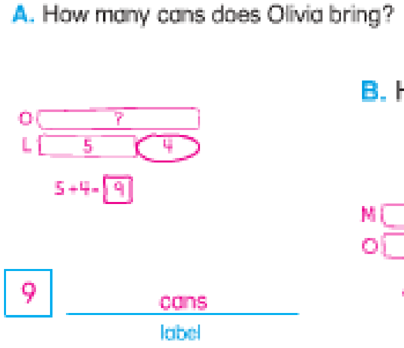
- Pay attention to the situation expressed in the problem
  - Not numbers and words/phrases
- What information does the problem ask for?
  - The number of cans Matt brings
- Paraphrase in their own words
- What is the hidden question?
  - How many cans does Olivia bring?
- Paraphrase again.

How many cans does Olivia bring?



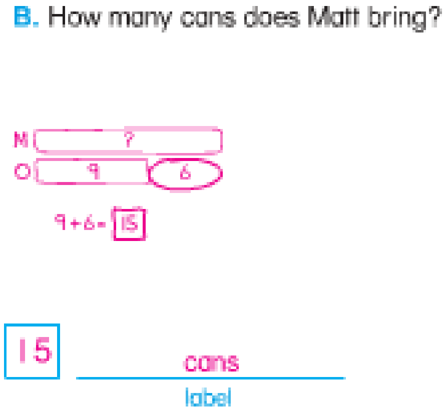
1. ~~Lindsay brings in 5 cans for the school food drive.~~ ~~Olivia brings in 4 more cans than Lindsay~~. Matt brings in 6 more cans than Olivia. How many cans does Matt bring?

Olivia's 9 cans.



A. How many cans does Olivia bring?

9 cans
label



B. How many cans does Matt bring?

15 cans
label

MULTI-STEP WORD PROBLEMS

- ~~Isabel~~ brought 36 pieces of fruit ~~for her soccer team. There are 16 apples, 12 bananas, a~~ ~~d the~~ rest are pears. How many ~~pieces of fruit~~ ~~e~~ pears?

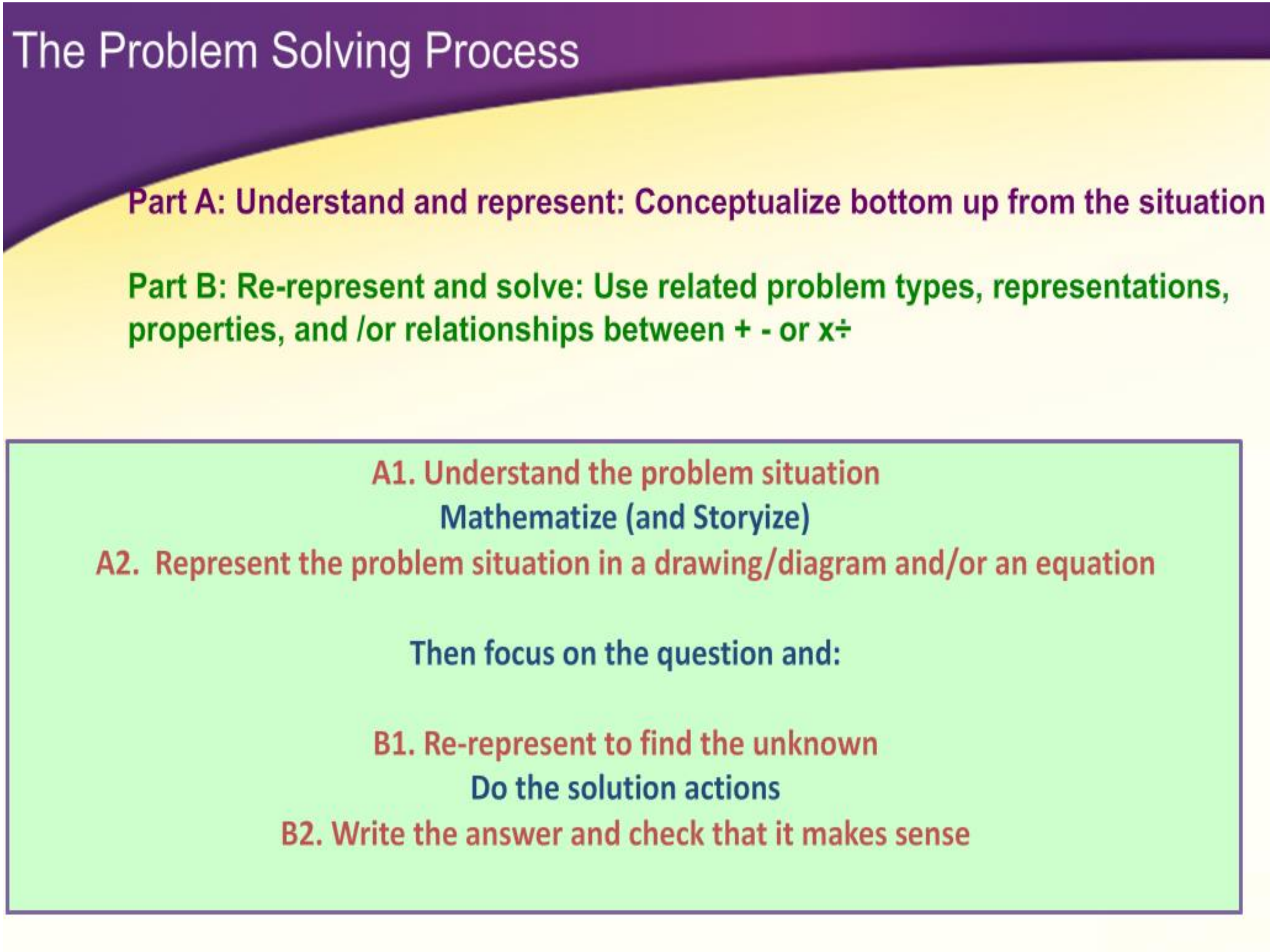
28 fruit

How many apples and bananas?
16 + 12

- What is the problem about?
  - Different kinds and amounts of fruit
- What do you need to find?
  - The pieces of fruit that are pears
- What do you know?
  - 16 + 12 = 28 fruit
- Mathematize
  - Rephrase into your own words
- Pay attention to the situation expressed in the problem
  - Not numbers and words/phrases

Situation: 36 = 28 + pears
Solve: 36 - 28 = 8 pears

PROBLEM SOLVING PROCESS



The Problem Solving Process

Part A: Understand and represent: Conceptualize bottom up from the situation

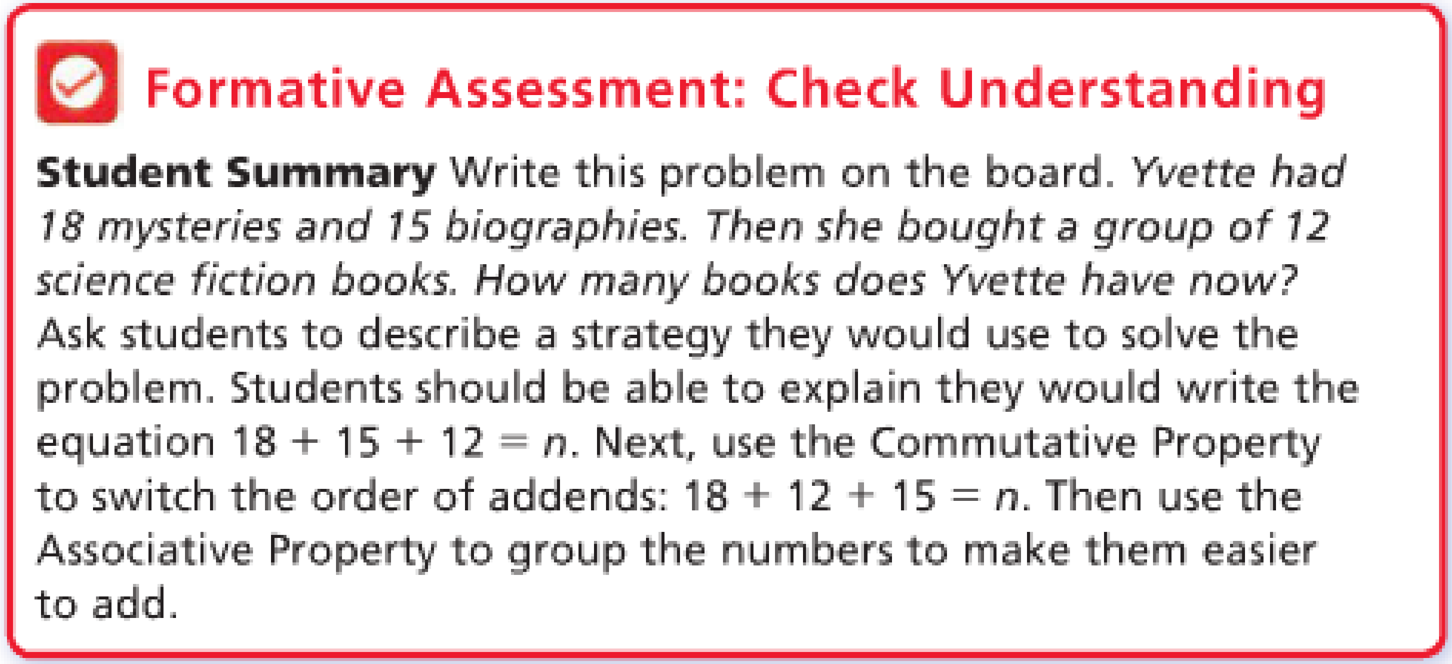
Part B: Re-represent and solve: Use related problem types, representations, properties, and /or relationships between + - or x÷

A1. Understand the problem situation
Mathematize (and Storyize)
A2. Represent the problem situation in a drawing/diagram and/or an equation

Then focus on the question and:

B1. Re-represent to find the unknown
Do the solution actions
B2. Write the answer and check that it makes sense

QUESTIONS/COMMENTS…



Formative Assessment: Check Understanding

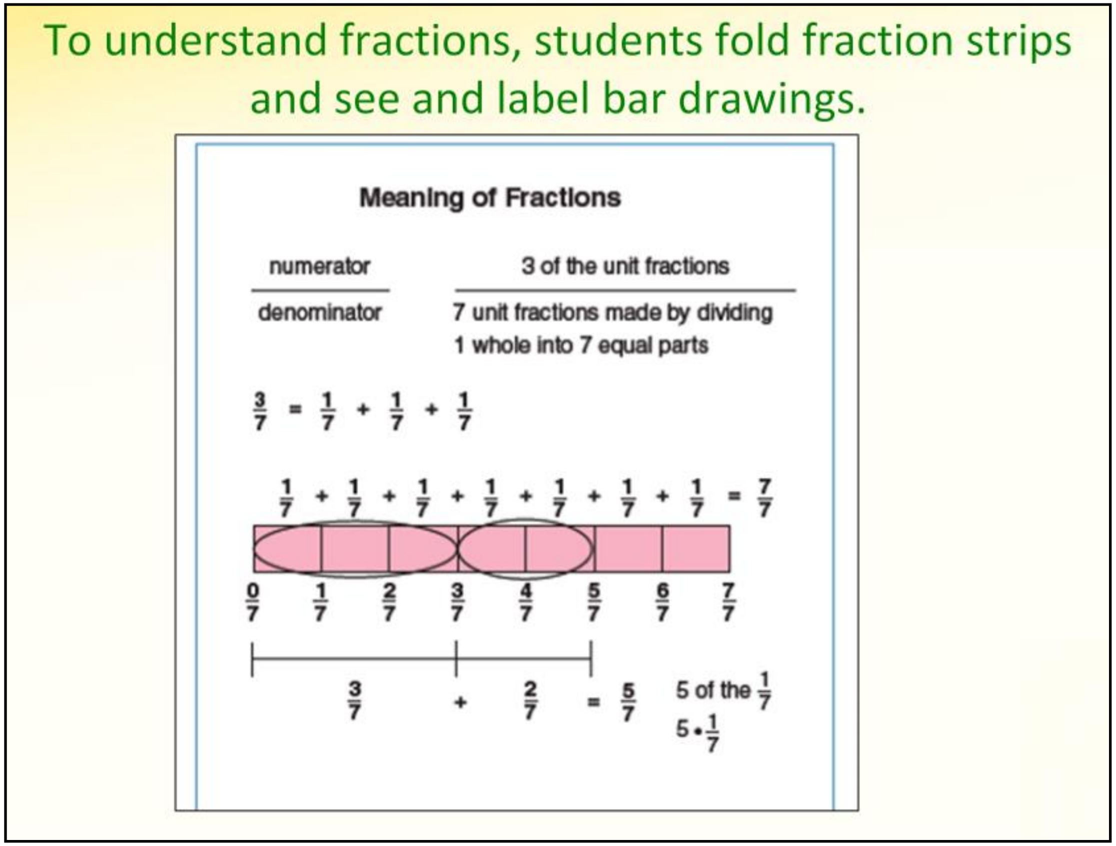
Student Summary Write this problem on the board. *Yvette had 18 mysteries and 15 biographies. Then she bought a group of 12 science fiction books. How many books does Yvette have now?* Ask students to describe a strategy they would use to solve the problem. Students should be able to explain they would write the equation $18 + 15 + 12 = n$. Next, use the Commutative Property to switch the order of addends: $18 + 12 + 15 = n$. Then use the Associative Property to group the numbers to make them easier to add.

FRACTIONS

- Create equivalent fractions by multiplying or dividing numerators and denominators of given fractions by the same number.
- Compare fractions using a variety of strategies, including rewriting them with a common denominator.
- Add and subtract fractions and mixed numbers with like and unlike denominators.

UNDERSTANDING FRACTIONS

- Emphasize that the equivalent fractions must have the same whole
  - Equation: composed of unit fractions
  - Visually: Bar diagram labeled with unit fractions
  - Visually: Relate bar diagram to Number line diagram



To understand fractions, students fold fraction strips and see and label bar drawings.

Meaning of Fractions

$$\frac{\text{numerator}}{\text{denominator}} \quad \frac{\text{3 of the unit fractions}}{\text{7 unit fractions made by dividing 1 whole into 7 equal parts}}$$

$$\frac{3}{7} = \frac{1}{7} + \frac{1}{7} + \frac{1}{7}$$

$$\frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{7}{7}$$

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

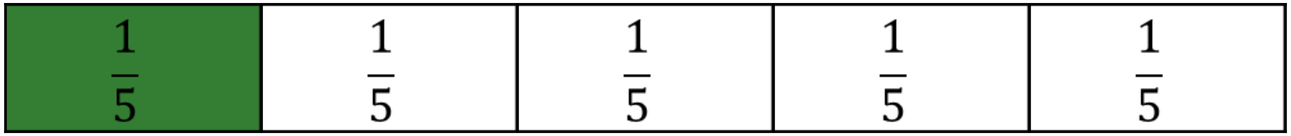
5 of the $\frac{1}{7}$

$5 \cdot \frac{1}{7}$

Seeing the unit fraction with **PLENTY** of ***visual representation & sense making*** students understand the denominator stays the same because it is just telling the name of the unit fraction.

UNIT FRACTIONS

- Understand unit fractions as the building blocks of fractions
  - Unit fraction $\frac{1}{n}$, where n is the number of equal parts the whole is divided into (MP abstractly/quantitatively)
    - 1/5 = 1 out of 5 equal parts



$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

- MathBoard fraction bars (MP tools)
  - Hands on "look" for understanding
  - Moving from hands on strips to pictures

UNIT FRACTIONS

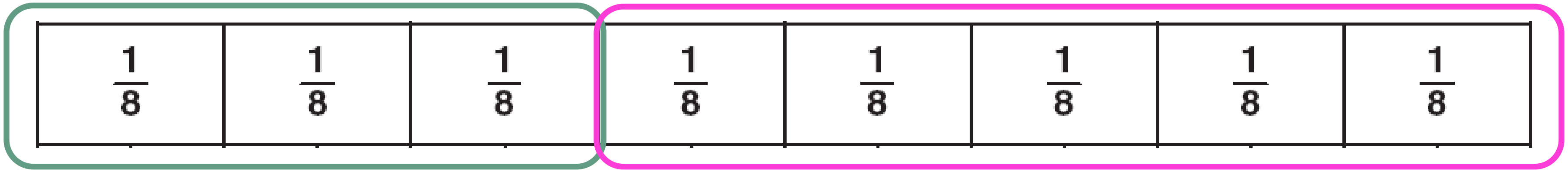
- Whole numbers are obtained by combining some number of 1
  - 3 = 1 + 1 + 1

- Same as fractions are obtained by combining some number of unit fractions

UNIT FRACTIONS

- Viewing non-unit fractions as sums of unit fractions helps students avoid common errors in adding fractions. (MP model/make sense)

3/8 + 5/8 = 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 + 1/8 = 8/8



UNIT FRACTIONS

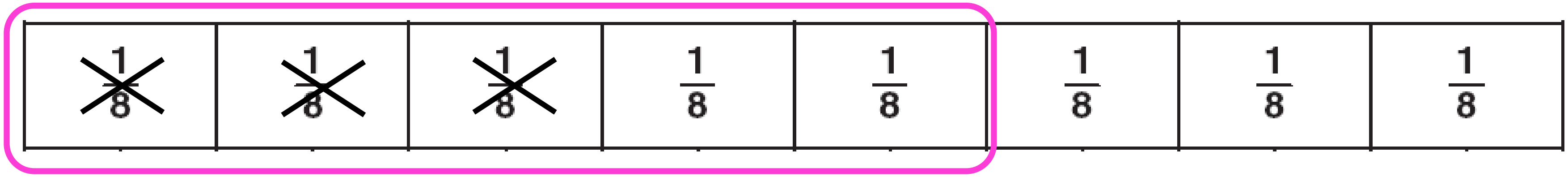
Viewing non-unit fractions as sums of unit fractions helps students avoid common errors in subtracting fractions.

- We had 5/8 of a pizza. Then we ate 3/8 of it. How much pizza is left?
  - What operation do we use?
    - We use subtraction because we are taking away one part from another.
  - How can we subtract the fractions?
    - Subtract the numerators and leave the denominators the same.

- Using the Mathboards and unit fractions show why we subtract fractions this way.

5/8 – 3/8 = 2/8

~~1/8~~ + ~~1/8~~ + ~~1/8~~ + 1/8 + 1/8



GENERATE EQUIVALENT HALVES

New vocabulary: n-split (or n-fracture)

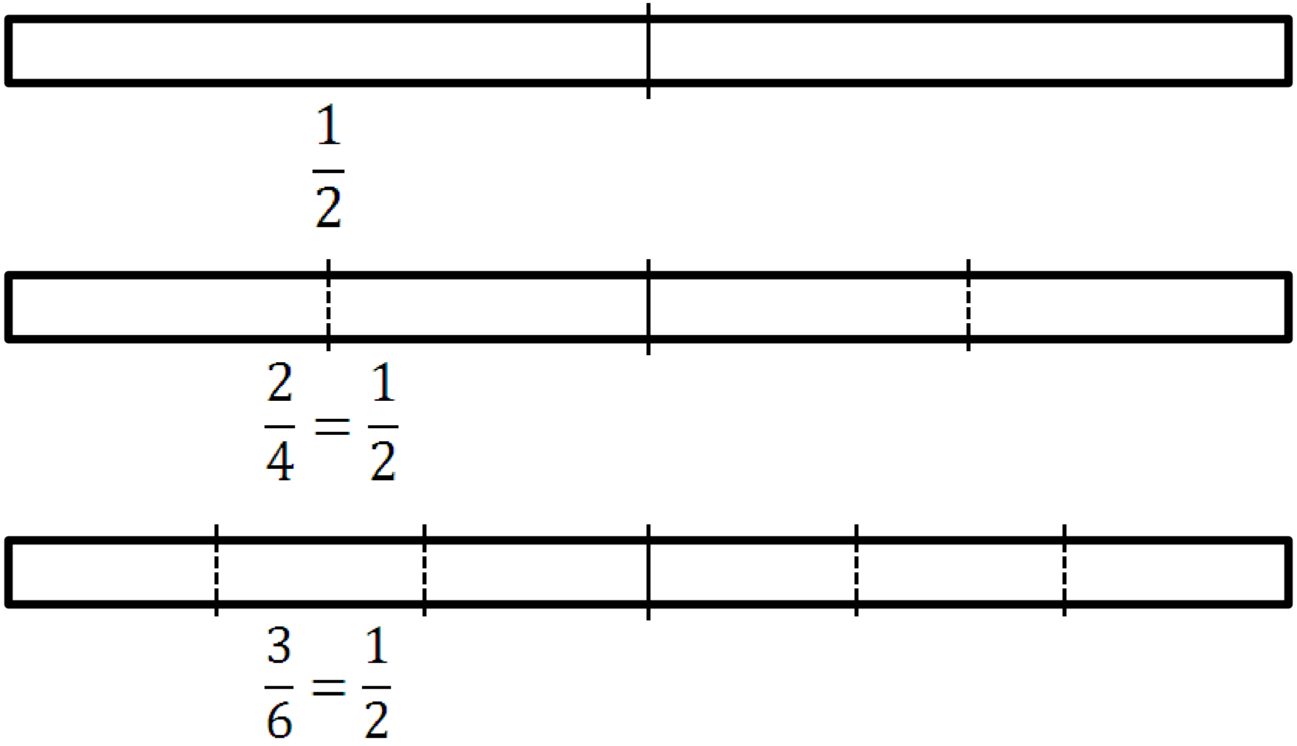
- Write the fraction ½ on the board. Ask students to suggest as many fractions as they can that are equivalent to ½.

$$\frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{2}{4}$$

- Please notice that the fraction chain does not need to be in the traditional order that we are used to seeing it.
- Leave this fraction chain on the board as we explore n-split

EQUIVALENT FRACTIONS

- Begin by looking at the fraction bars that are equal to1/2
- Draw a vertical line at the end of 1/2
- Discuss the relationships the students should see
  - Use the Mathboards (MP tools)
  - Look for the relationships when finding all of the ½ fractions (MP structure)



$$\frac{1}{2}$$

$$\frac{2}{4} = \frac{1}{2}$$

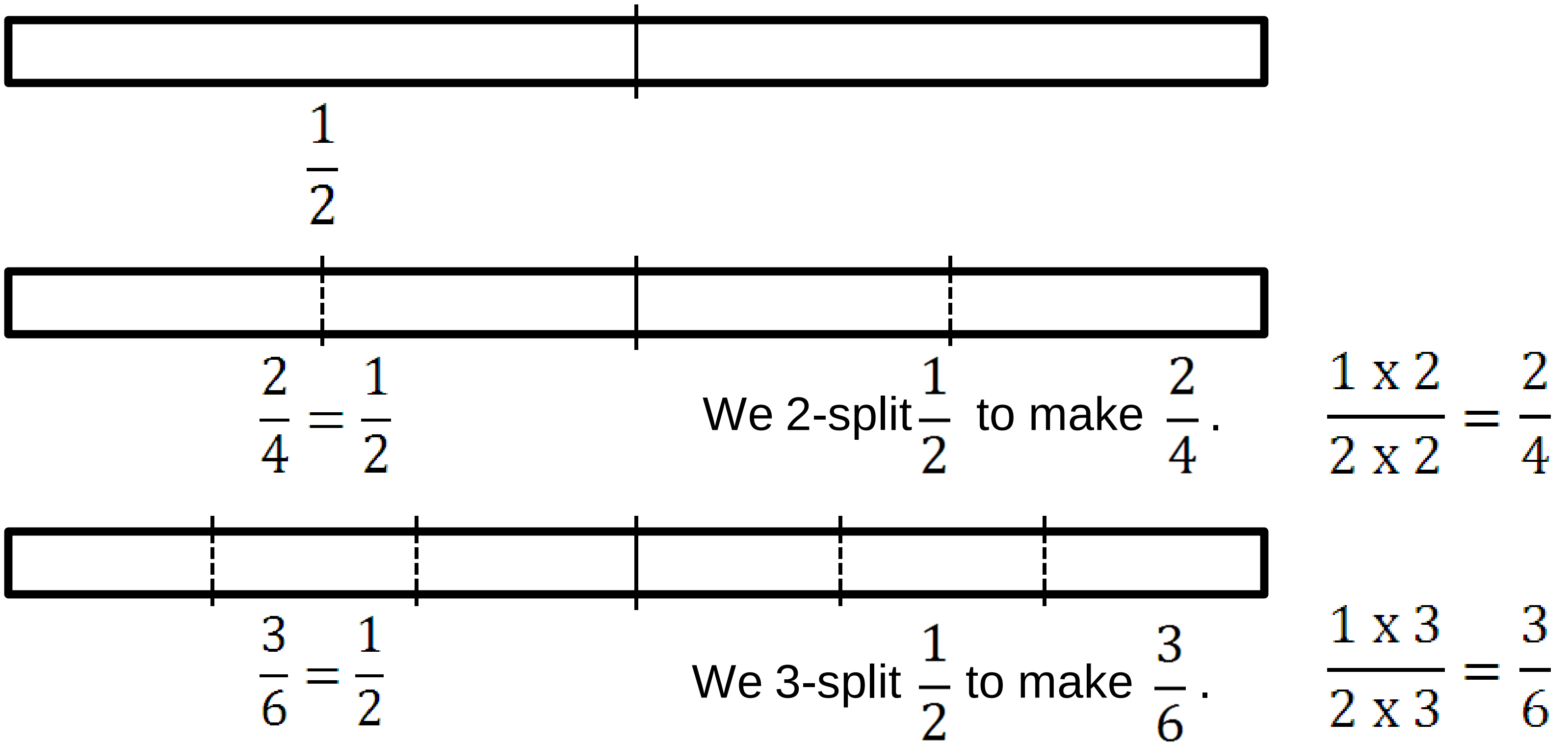
$$\frac{3}{6} = \frac{1}{2}$$

EQUIVALENT FRACTIONS: I DO

- Show the dividing of the ½ length into smaller unit fractions and the multiplying of the number of unit fractions to make equivalent fractions (the same size part of the whole) (MP abstract/quantity)

Common error to draw 1 too many vertical lines (2-split, draw 2 lines not 1)

  - What does it mean to multiply the numerator and denominator of ½ by the same number?



$\frac{1}{2}$

$\frac{2}{4} = \frac{1}{2}$ We 2-split $\frac{1}{2}$ to make $\frac{2}{4}$. $\frac{1 \text{ x } 2}{2 \text{ x } 2} = \frac{2}{4}$

$\frac{3}{6} = \frac{1}{2}$ We 3-split $\frac{1}{2}$ to make $\frac{3}{6}$. $\frac{1 \text{ x } 3}{2 \text{ x } 3} = \frac{3}{6}$

EQUIVALENT FRACTIONS: WE/YOU DO

- Math Talk
  - What kind of n-split would create this fraction?
  - What would you have to multiply the top and bottom by to get that fraction?
  - Would this work for any multiplier?
- You Try!
  - What kind of n-split would create this fraction?
  - What would you have to multiply the top and bottom by to get that fraction?

$$\frac{1}{2} = \frac{8}{16}$$

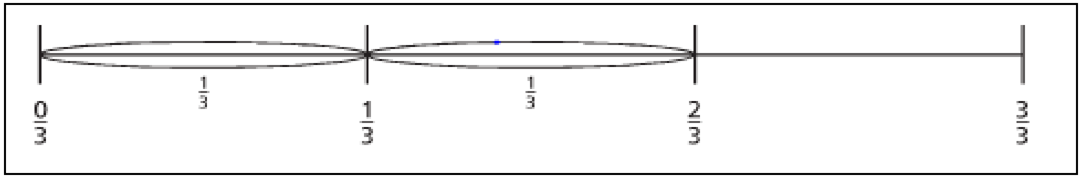
We 8-split $\frac{1}{2}$ to make $\frac{8}{16}$. $\frac{1 \times 8}{2 \times 8} = \frac{8}{16}$

$$\frac{1}{2} = \frac{100}{200}$$

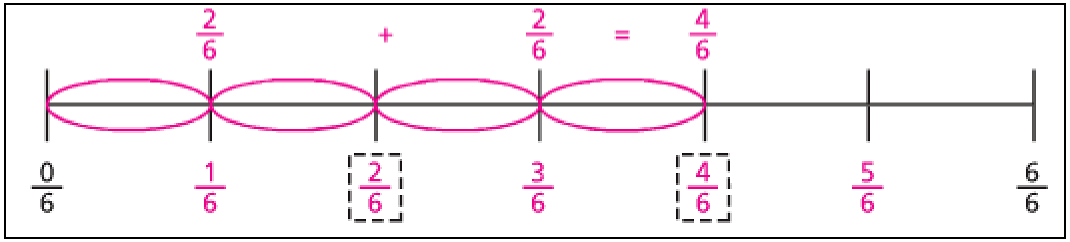
We 100-split $\frac{1}{2}$ to make $\frac{100}{200}$. $\frac{1 \times 100}{2 \times 100} = \frac{100}{200}$

FRACTIONS ON NUMBER LINES

- Find equivalent fractions by multiplying
  - Finding equivalent fractions for 2/3
    - What is the total of the circled thirds?
      - $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

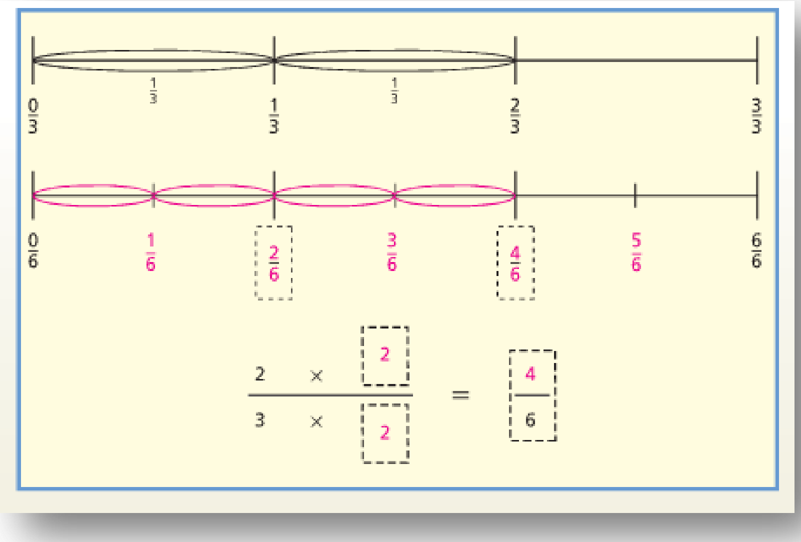


- Label $\frac{1}{6}$, notice the fractions in the boxes are equivalent to $\frac{1}{3}$ and $\frac{2}{3}$.
- Circle enough sixths to make $\frac{1}{3}$ and $\frac{2}{3}$. Then write the total about each part.



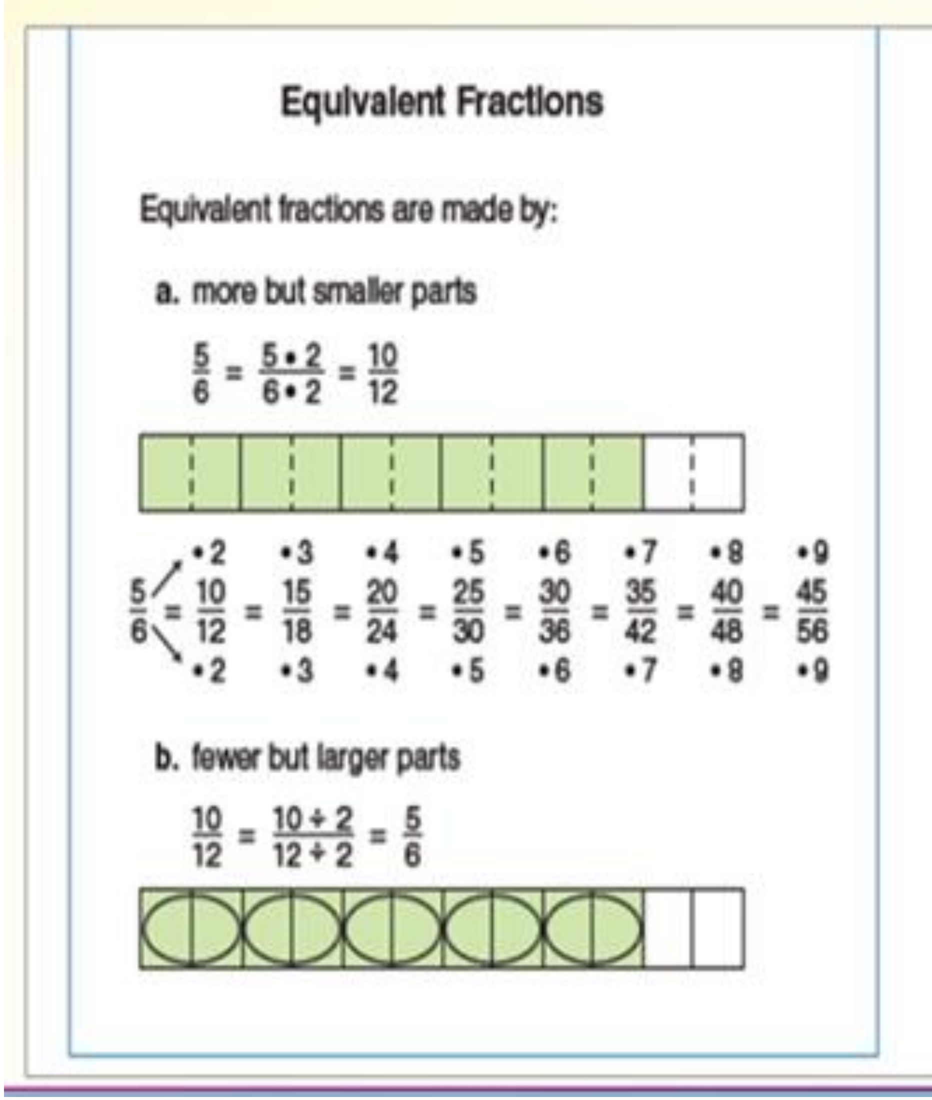
MATH TALK

- Using the number line as a tool to provide structure…
- How many sixths does it take to make $\frac{1}{3}$?
- How many sixths does it take to make $\frac{2}{3}$?
- How can $\frac{4}{6}$ be equal to $\frac{2}{3}$ when $\frac{4}{6}$ has greater numbers than $\frac{2}{3}$?



EQUIVALENT FRACTIONS

- Equivalent fractions are made by:
  - **More** but **smaller** parts
    - $\frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$
  - **Fewer** but **larger** parts
    - $\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$



Equivalent Fractions

Equivalent fractions are made by:

a. more but smaller parts

$$\frac{5}{6} = \frac{5 \bullet 2}{6 \bullet 2} = \frac{10}{12}$$

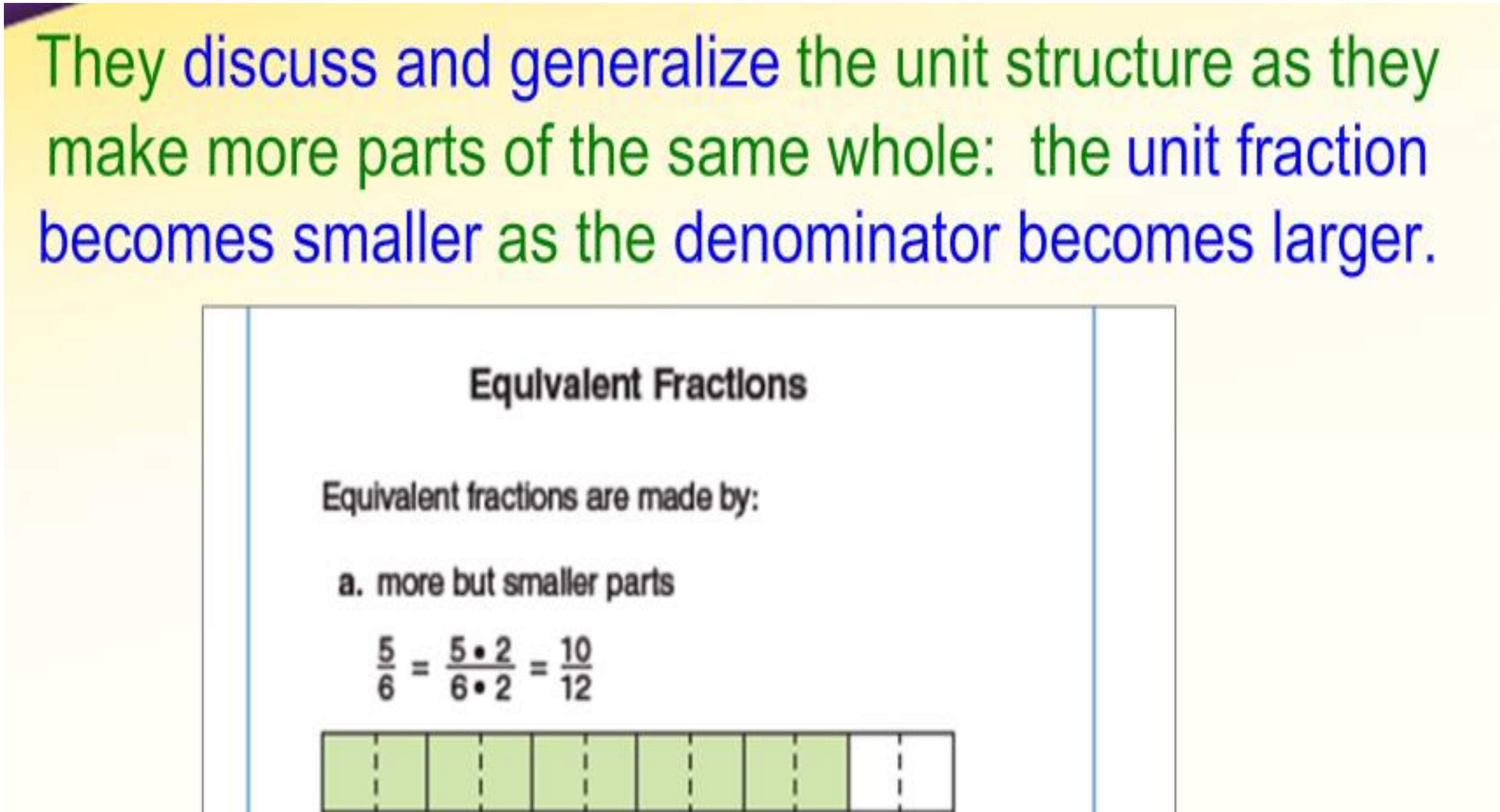
$$\frac{5}{6} = \overset{\bullet 2}{\underset{\bullet 2}{\frac{10}{12}}} = \overset{\bullet 3}{\underset{\bullet 3}{\frac{15}{18}}} = \overset{\bullet 4}{\underset{\bullet 4}{\frac{20}{24}}} = \overset{\bullet 5}{\underset{\bullet 5}{\frac{25}{30}}} = \overset{\bullet 6}{\underset{\bullet 6}{\frac{30}{36}}} = \overset{\bullet 7}{\underset{\bullet 7}{\frac{35}{42}}} = \overset{\bullet 8}{\underset{\bullet 8}{\frac{40}{48}}} = \overset{\bullet 9}{\underset{\bullet 9}{\frac{45}{56}}}$$

b. fewer but larger parts

$$\frac{10}{12} = \frac{10 \div 2}{12 \div 2} = \frac{5}{6}$$

GENERALIZE UNIT STRUCTURE

- As you make more parts of the same whole, the unit fraction becomes smaller
  - Denominator becomes larger 5/6 becomes 10/12
  - Each unit fraction 1/6 is divided into 2 equal parts
    - There will be 2 equal parts for each 1 part so you get 10/12



They discuss and generalize the unit structure as they make more parts of the same whole: the unit fraction becomes smaller as the denominator becomes larger.

Equivalent Fractions

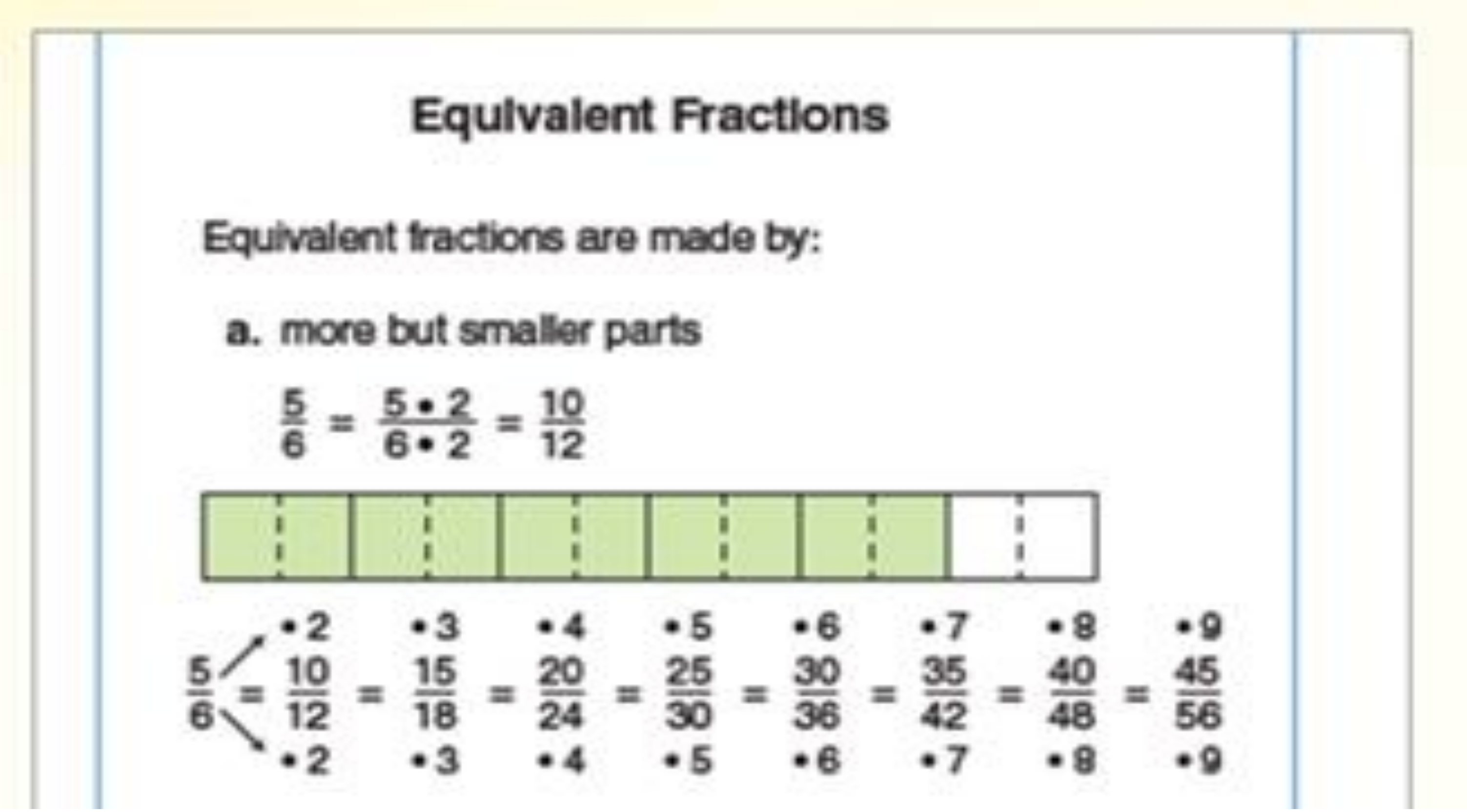
Equivalent fractions are made by:

a. more but smaller parts

$$\frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$$

GENERALIZE UNIT STRUCTURE

- Multiplication table
  - The 5 row and the 6 row helps students see there are many equivalent fractions made by multiplying another fraction by the same number on the top and bottom.



Equivalent Fractions

Equivalent fractions are made by:

a. more but smaller parts

$$\frac{5}{6} = \frac{5 \cdot 2}{6 \cdot 2} = \frac{10}{12}$$

$$\frac{5}{6} = \overset{\cdot 2}{\underset{\cdot 2}{\frac{10}{12}}} = \overset{\cdot 3}{\underset{\cdot 3}{\frac{15}{18}}} = \overset{\cdot 4}{\underset{\cdot 4}{\frac{20}{24}}} = \overset{\cdot 5}{\underset{\cdot 5}{\frac{25}{30}}} = \overset{\cdot 6}{\underset{\cdot 6}{\frac{30}{36}}} = \overset{\cdot 7}{\underset{\cdot 7}{\frac{35}{42}}} = \overset{\cdot 8}{\underset{\cdot 8}{\frac{40}{48}}} = \overset{\cdot 9}{\underset{\cdot 9}{\frac{45}{56}}}$$

EQUIVALENCE IN THE MULTIPLICATION TABLE

- *What does row 3 in each table show?*
    - *Multiples of 3*
- *What does row 5 in each table show*
    - *Multiples of 5*

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70

Students think of the numbers in row 3 as numerators and the numbers in row 5 as denominators.

- *Why is 3/5 equivalent to 6/10?*
    - *Both the numerator and denominator of 3/5 have been multiplied by 2.*

MATH TALK

The first fraction 3/5, is the simplest fraction.

- *Why can't we write it with smaller numbers for the numerator and denominator?*

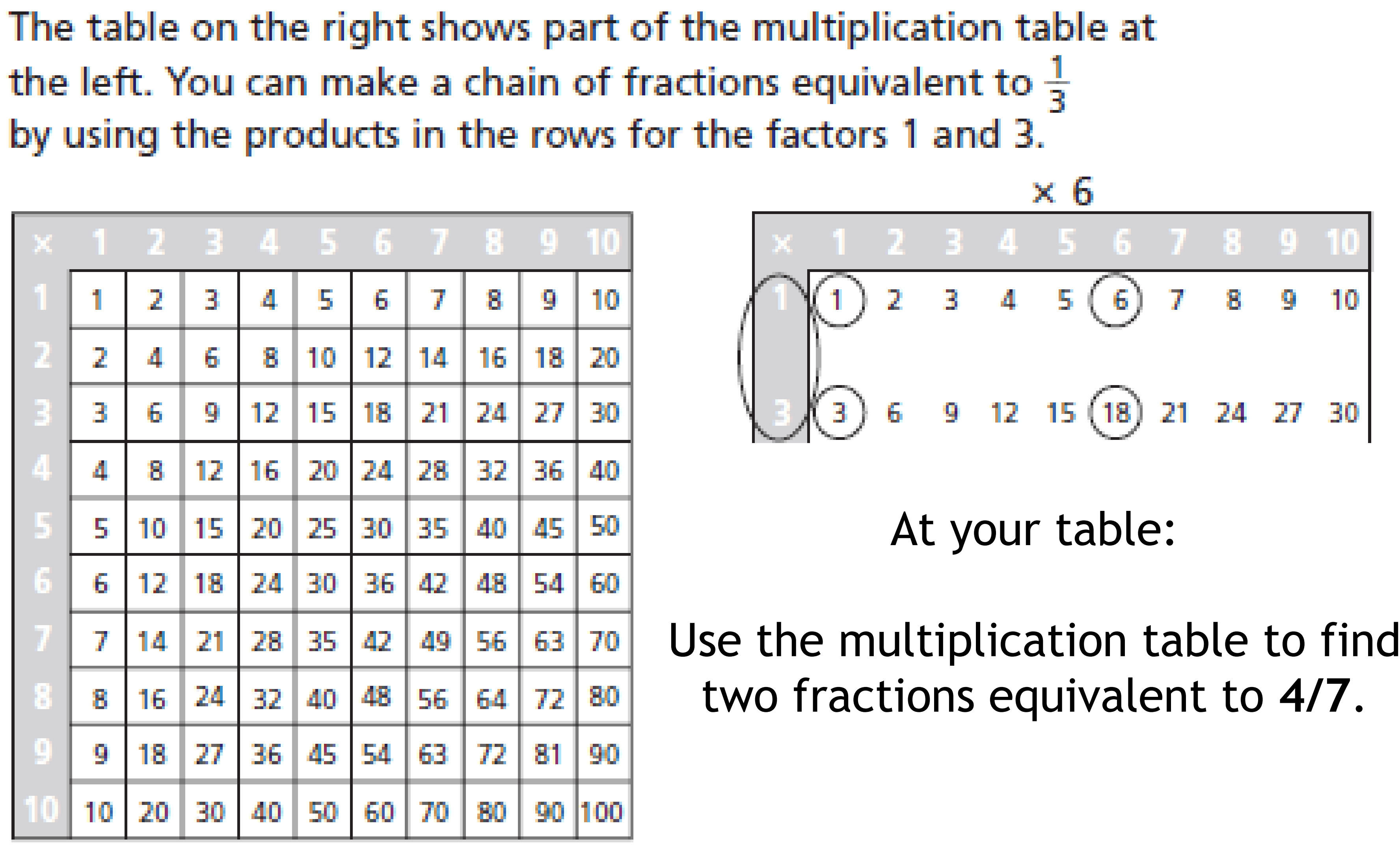
- *Where do you see the multipliers in the table for the fractions?*

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70

Simplify Dividing the numerator and denominator by the same number makes the fraction smaller by making larger unit fractions.

Unsimplify Multiplying the numerator and denominator by the same number makes the fraction smaller by making smaller unit fractions.

USE A MULTIPLICATION TABLE



The table on the right shows part of the multiplication table at the left. You can make a chain of fractions equivalent to $\frac{1}{3}$ by using the products in the rows for the factors 1 and 3.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

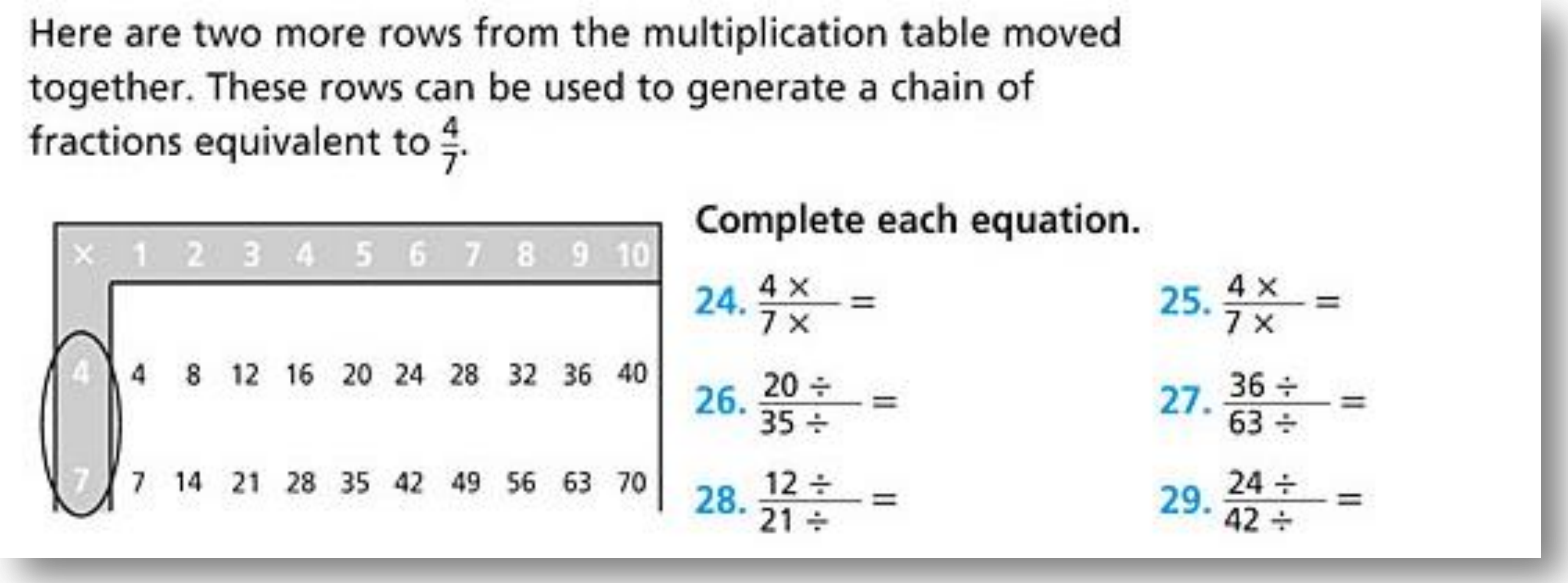
× 6

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
3	3	6	9	12	15	18	21	24	27	30

At your table:

Use the multiplication table to find two fractions equivalent to **4/7**.

USE A MULTIPLICATION TABLE



Here are two more rows from the multiplication table moved together. These rows can be used to generate a chain of fractions equivalent to $\frac{4}{7}$.

×	1	2	3	4	5	6	7	8	9	10
4	4	8	12	16	20	24	28	32	36	40
7	7	14	21	28	35	42	49	56	63	70

Complete each equation.

24. $\frac{4 \times}{7 \times} =$

25. $\frac{4 \times}{7 \times} =$

26. $\frac{20 \div}{35 \div} =$

27. $\frac{36 \div}{63 \div} =$

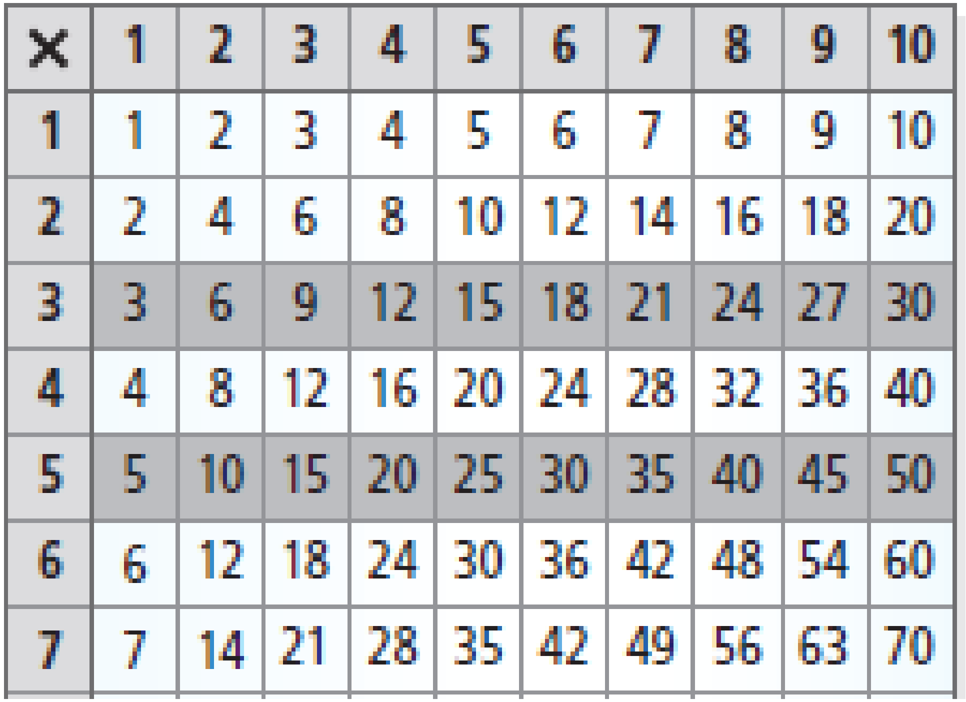
28. $\frac{12 \div}{21 \div} =$

29. $\frac{24 \div}{42 \div} =$

- Conceptualize why you can multiply the numerator and denominator by forms of 1 to find equivalent fractions
- Connect understanding of fraction bar models to the multiplication table

MATH TALK

- *Building structure within mathematics.*
- *How can you change* $\frac{3}{5}$ *to* $\frac{18}{30}$*?*
- *How can you simplify* $\frac{18}{30}$*?*
- *How can you change* $\frac{3}{5}$ *to* $\frac{27}{45}$*?*

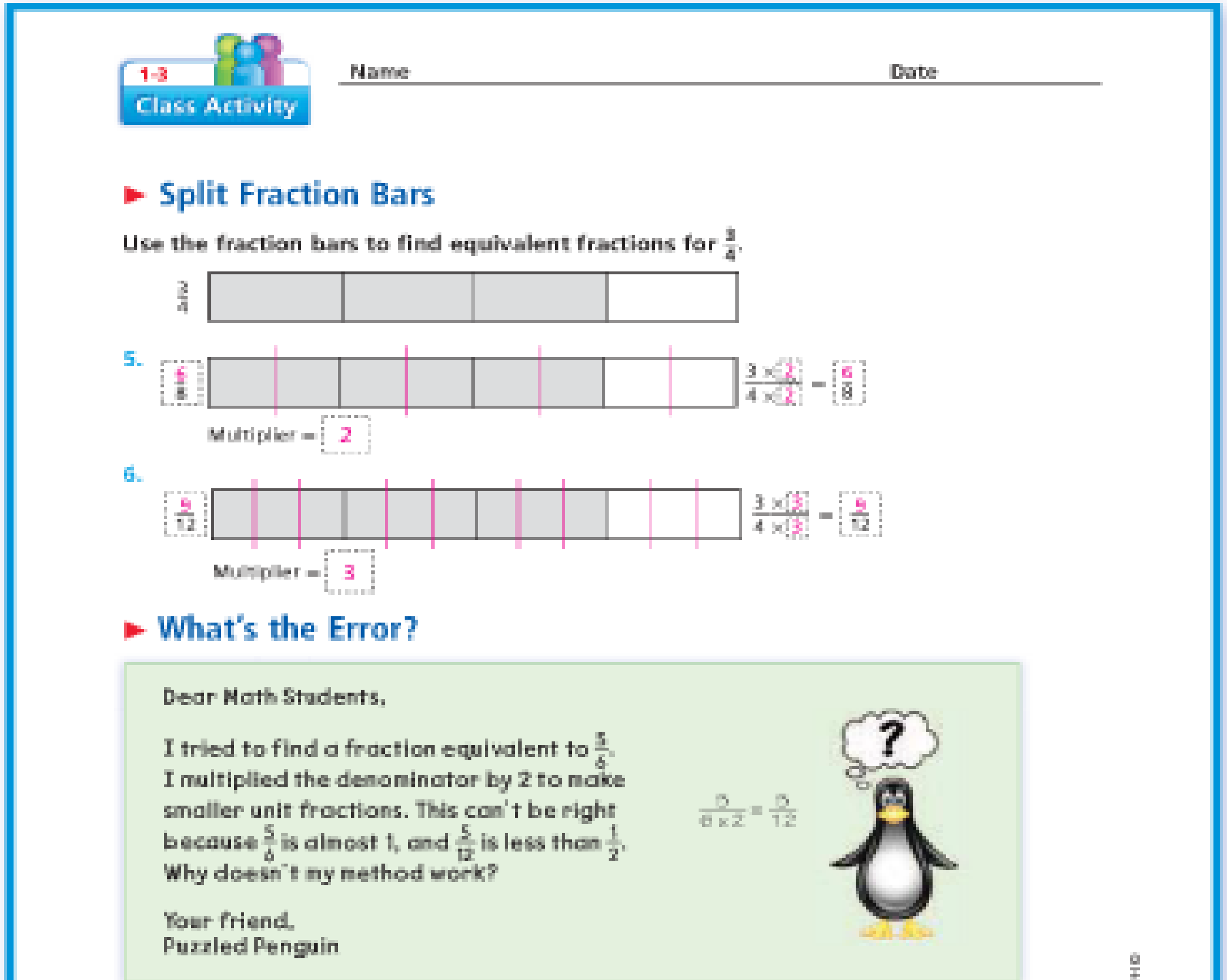


×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70

MATH TALK

Do and discuss Class Activity

- What's the error? (MP viable arguments/reasoning)



1-8 Class Activity

Name Date

Split Fraction Bars

Use the fraction bars to find equivalent fractions for $\frac{3}{4}$.

$\frac{3}{4}$

5. $\frac{6}{8}$ $\frac{3 \times 2}{4 \times 2} = \frac{6}{8}$

Multiplier = 2

6. $\frac{9}{12}$ $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

Multiplier = 3

What's the Error?

Dear Math Students,

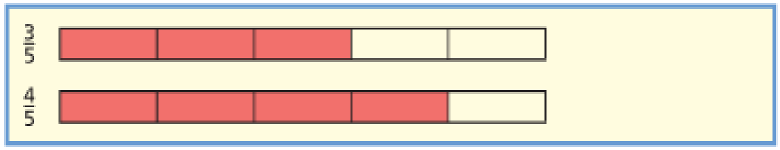
I tried to find a fraction equivalent to $\frac{5}{6}$. I multiplied the denominator by 2 to make smaller unit fractions. This can't be right because $\frac{5}{6}$ is almost 1, and $\frac{5}{12}$ is less than $\frac{1}{2}$. Why doesn't my method work?

$\frac{5}{6 \times 2} = \frac{5}{12}$

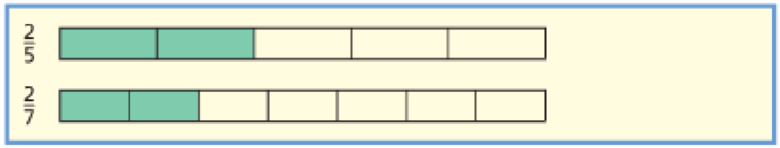
Your friend,
Puzzled Penguin

COMPARING FRACTIONS

- Reasoning
  - Understanding like denominators
    - Fraction with the greater numerator-- is visually larger and therefore the greater fraction

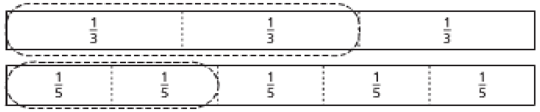


  - Understanding like numerator
    - Fraction with the lesser denominator – is visually larger and therefore the greater fraction

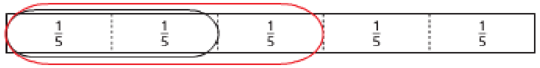


STRATEGIES TO COMPARE

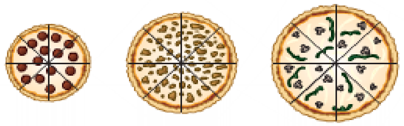
- Number lines & Fraction bars
  - Compare
  - Explore fraction benchmarks
  - Equivalent fractions

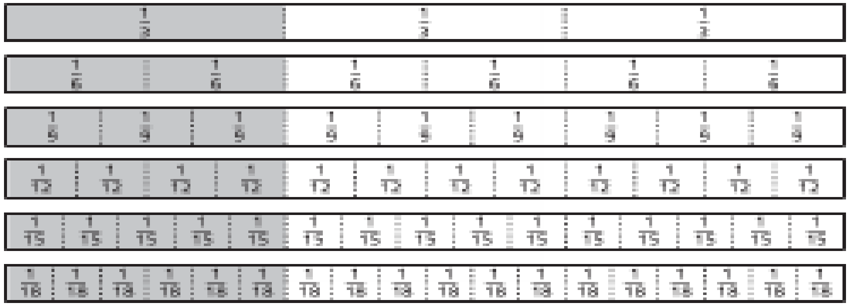


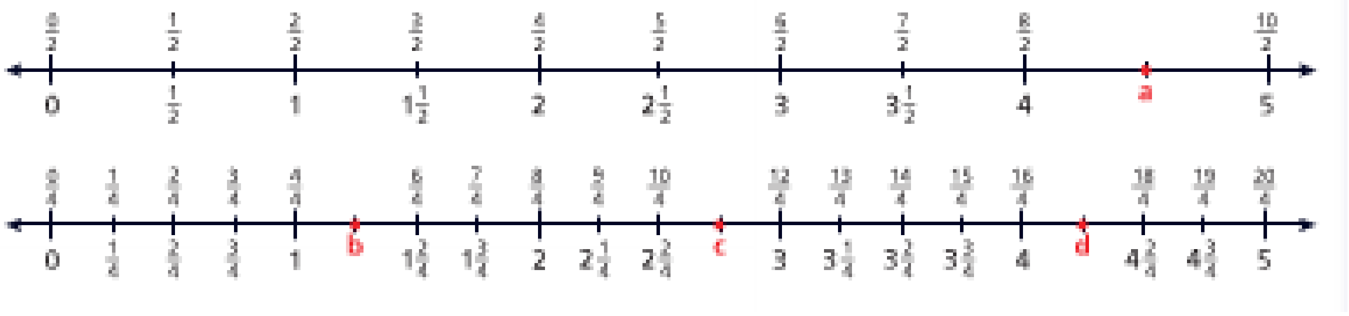
Fraction bars are also used to help students compare fractions with different numerators and the same denominator. This model shows that $\frac{2}{5} < \frac{3}{5}$.



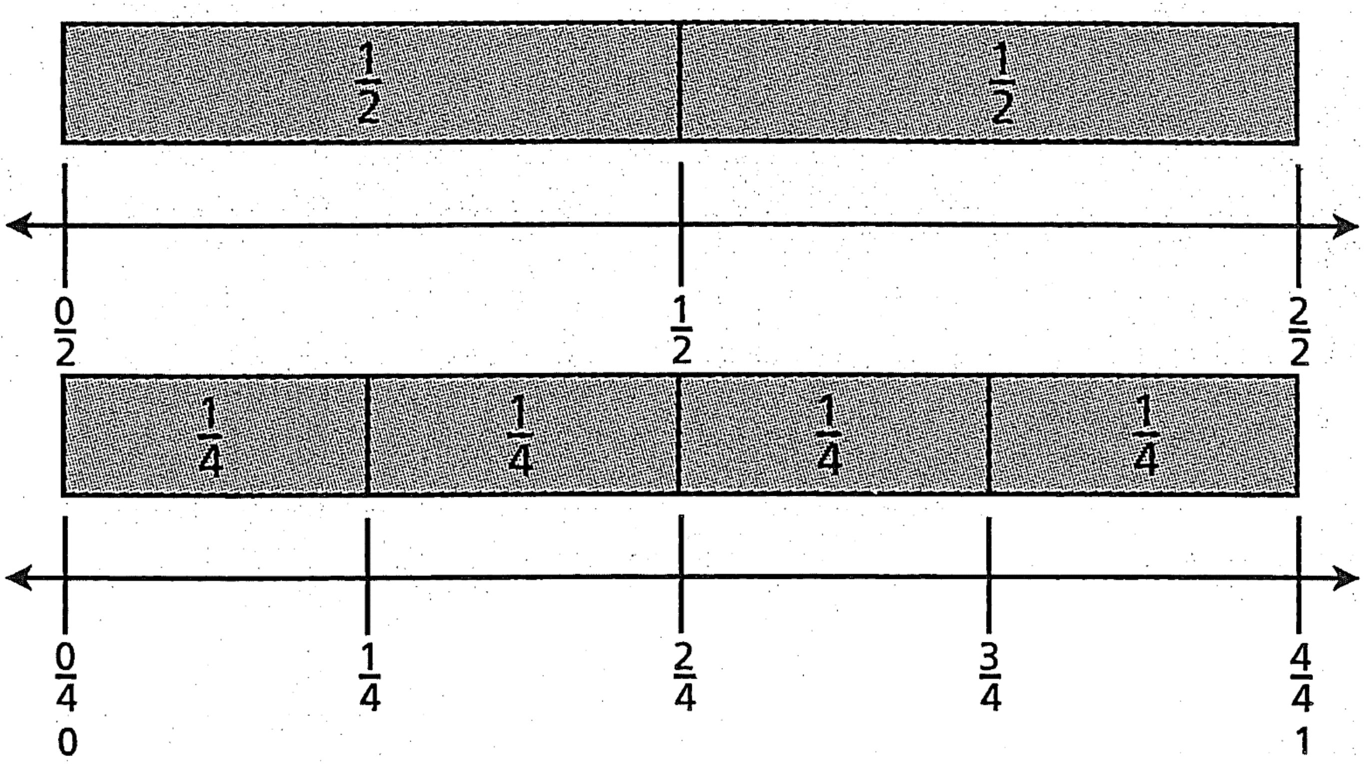
Students also explore comparing fractions of different-sized wholes. Models help them visualize that, for example, $\frac{1}{8}$ of a bigger whole is greater than $\frac{1}{8}$ of a smaller whole.





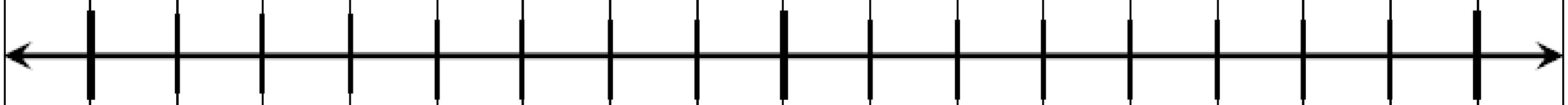


FRACTIONS ON A NUMBER LINE



- How are number lines similar, yet different from fraction strips?

FRACTION LINE-UP



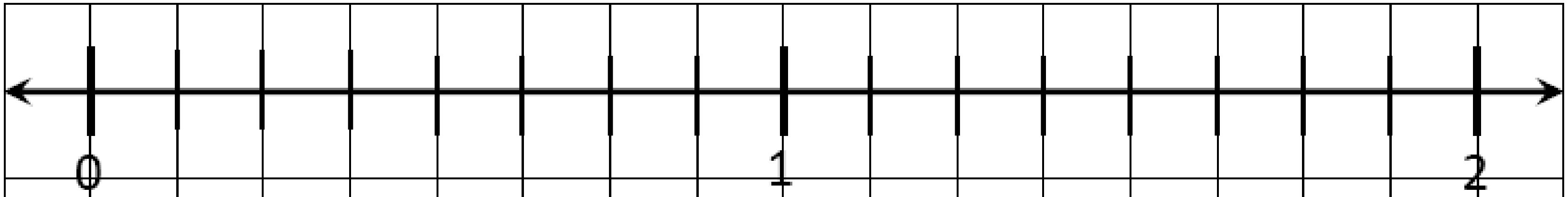
	0								1								2
Halves:	0/2				1/2				1				1 1/2				2
Fourths:	0/4		1/4		2/4		3/4		1		1 1/4		1 2/4		1 3/4		2
Eighths:	**0/8**	**1/8**	**2/8**	**3/8**	**4/8**	**5/8**	**6/8**	**7/8**	**1**	**1 1/8**	**1 2/8**	**1 3/8**	**1 4/8**	**1 5/8**	**1 6/8**	**1 7/8**	**2**

Mark and label the halves on the number line.

Mark and label the fourths on the number line.

Mark and label the eighths on the number line.

FRACTION LINE-UP
NUMERATOR LARGER THAN DENOMINATOR



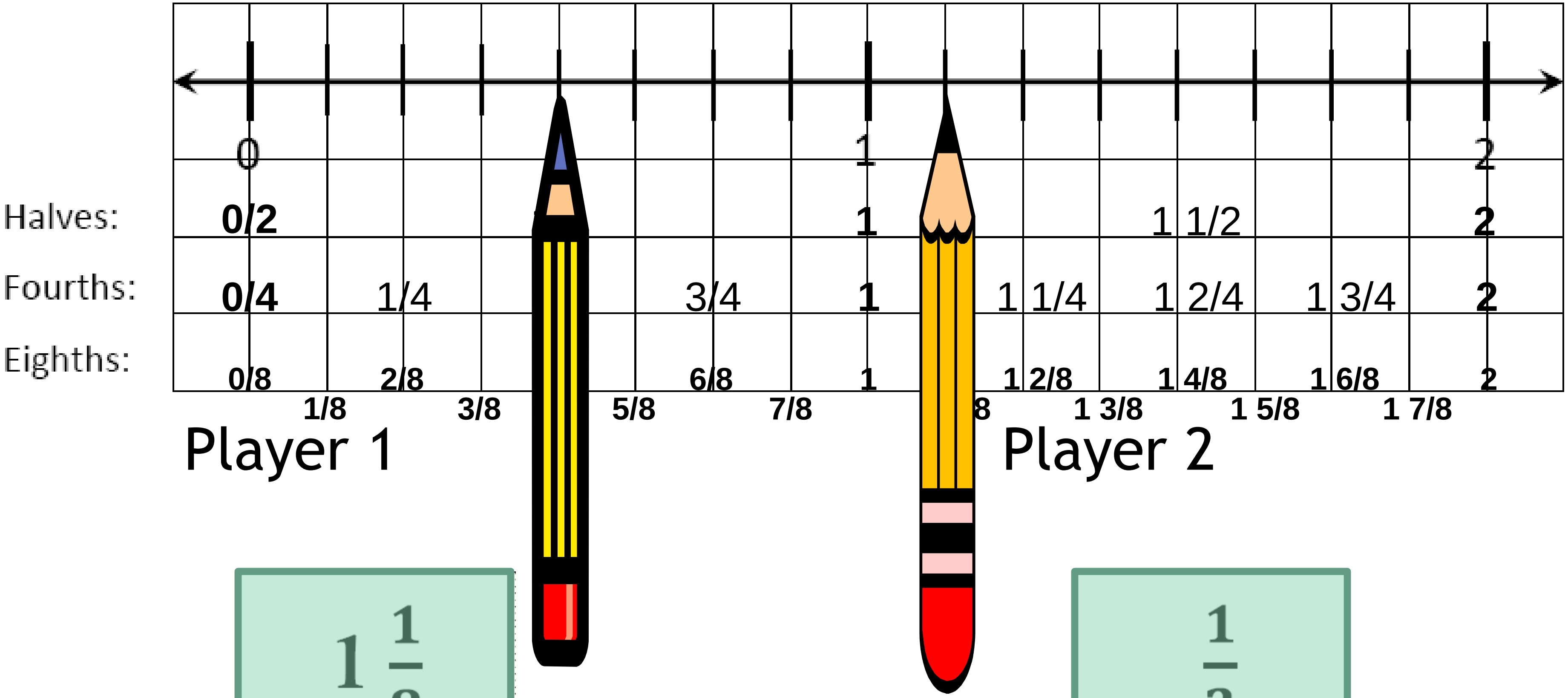
Halves:	0/2				1/2				2/2				3/2				4/2
Fourths:	0/4		1/4		2/4		3/4		4/4		5/4		6/4		7/4		8/4
Eighths:	**0/8**	**1/8**	**2/8**	**3/8**	**4/8**	**5/8**	**6/8**	**7/8**	**8/8**	**9/8**	**10/8**	**11/8**	**12/8**	**13/8**	**14/8**	**15/8**	**16/8**

Mark and label the halves on the number line.

Mark and label the fourths on the number line.

Mark and label the eighths on the number line.

CLOSEST TO ONE!



Player 1

Player 2

$1\frac{1}{8}$

$\frac{1}{2}$

“I have one and one-eighth”

“One and one-eighth is closer to one”

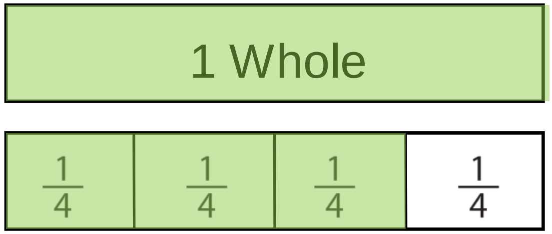
“I have one half”

FRACTION G.O. SHEETS

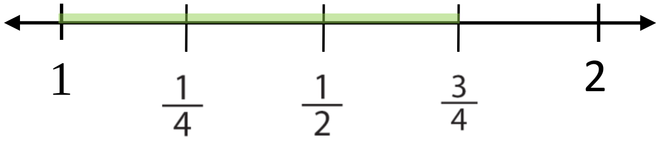
Fraction

$$1\frac{3}{4}$$

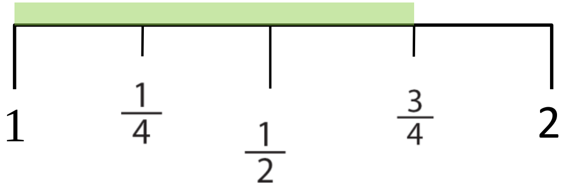
Fraction Strip



Number Line



Ruler



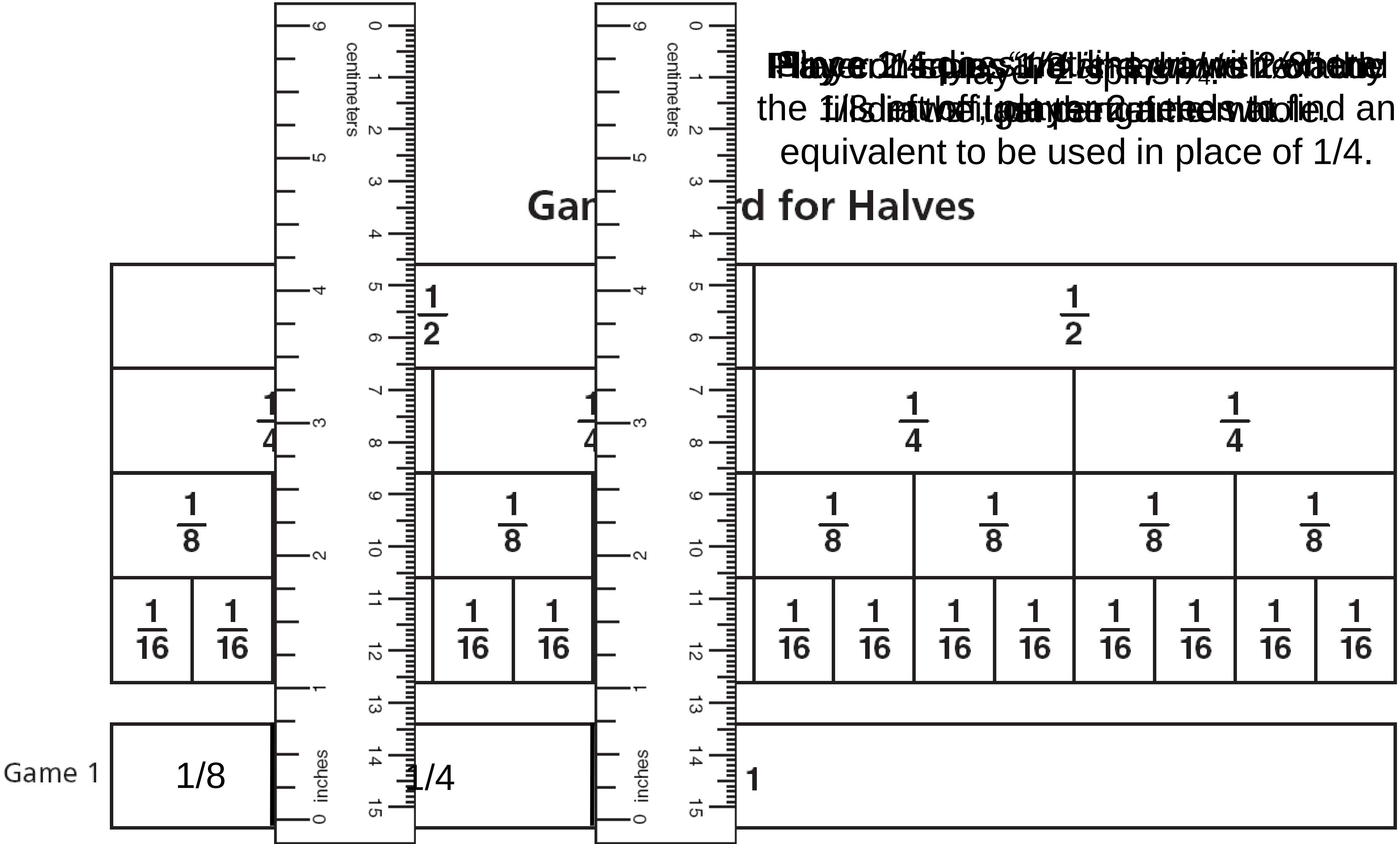
SPINNING A WHOLE

- Please take out the directions, game spinner, game mat and a paper clip to use with your pencil as a spinner.
- Please read the directions and discuss the objective of the game.
  - Do any steps need clarifying?
  - Note: We will play on a single game board in order to increase the opportunity for “math talk” during the game.
- Please do not begin playing. We will discuss an example before we play a game on our own.

SPINNING A WHOLE

[illegible] Player 2 spins 1/4. [illegible]

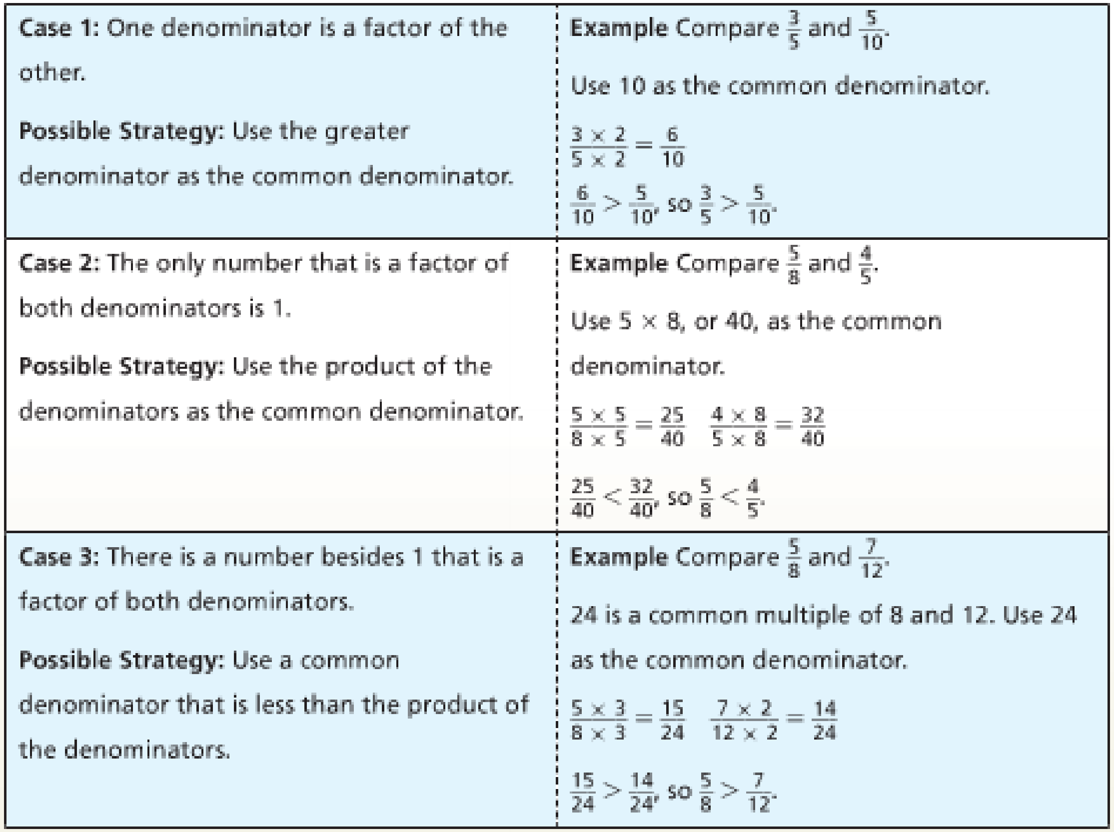
the [illegible] an
equivalent to be used in place of 1/4.



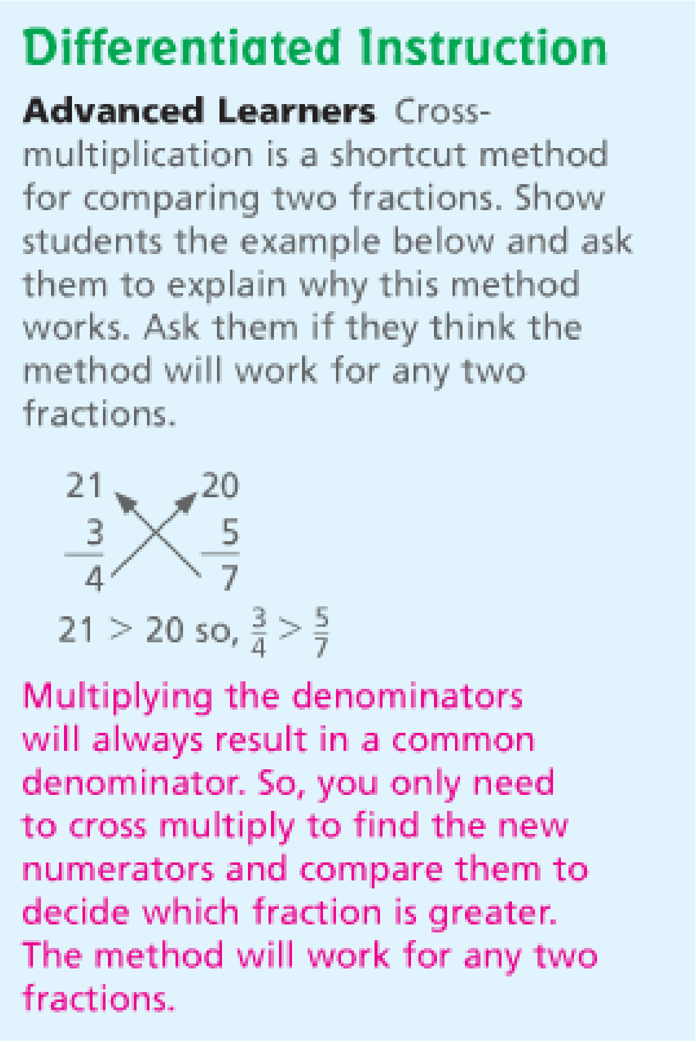
COMPARING FRACTIONS

Advanced because they should understand the mathematical reasons this works

- Unlike denominators
  - Need to find common denominators



Case 1: One denominator is a factor of the other. **Possible Strategy:** Use the greater denominator as the common denominator.	**Example** Compare $\frac{3}{5}$ and $\frac{5}{10}$. Use 10 as the common denominator. $\frac{3 \times 2}{5 \times 2} = \frac{6}{10}$ $\frac{6}{10} > \frac{5}{10}$, so $\frac{3}{5} > \frac{5}{10}$.
Case 2: The only number that is a factor of both denominators is 1. **Possible Strategy:** Use the product of the denominators as the common denominator.	**Example** Compare $\frac{5}{8}$ and $\frac{4}{5}$. Use 5 × 8, or 40, as the common denominator. $\frac{5 \times 5}{8 \times 5} = \frac{25}{40}$ $\frac{4 \times 8}{5 \times 8} = \frac{32}{40}$ $\frac{25}{40} < \frac{32}{40}$, so $\frac{5}{8} < \frac{4}{5}$.
Case 3: There is a number besides 1 that is a factor of both denominators. **Possible Strategy:** Use a common denominator that is less than the product of the denominators.	**Example** Compare $\frac{5}{8}$ and $\frac{7}{12}$. 24 is a common multiple of 8 and 12. Use 24 as the common denominator. $\frac{5 \times 3}{8 \times 3} = \frac{15}{24}$ $\frac{7 \times 2}{12 \times 2} = \frac{14}{24}$ $\frac{15}{24} > \frac{14}{24}$, so $\frac{5}{8} > \frac{7}{12}$.



Differentiated Instruction

Advanced Learners Cross-multiplication is a shortcut method for comparing two fractions. Show students the example below and ask them to explain why this method works. Ask them if they think the method will work for any two fractions.

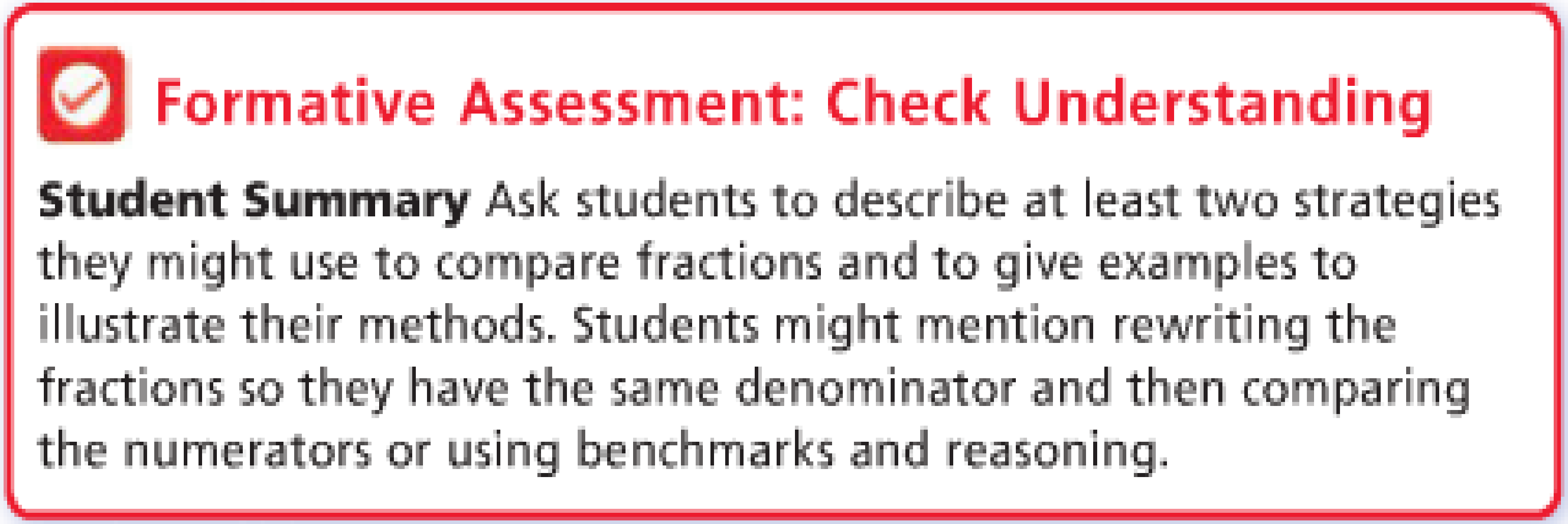
21 20

$\frac{3}{4}$ $\frac{5}{7}$

21 > 20 so, $\frac{3}{4} > \frac{5}{7}$

Multiplying the denominators will always result in a common denominator. So, you only need to cross multiply to find the new numerators and compare them to decide which fraction is greater. The method will work for any two fractions.

MATH TALK



Formative Assessment: Check Understanding

Student Summary Ask students to describe at least two strategies they might use to compare fractions and to give examples to illustrate their methods. Students might mention rewriting the fractions so they have the same denominator and then comparing the numerators or using benchmarks and reasoning.

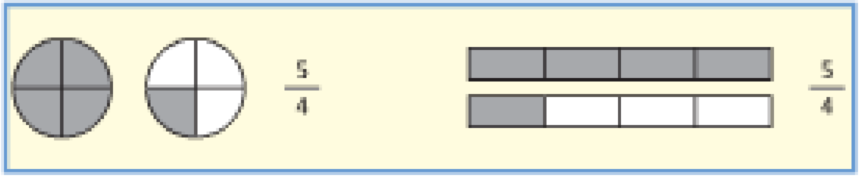
Which mathematical practices are being used to answer this question and why do you think that?

FRACTIONS GREATER THAN 1 AND MIXED NUMBERS

- Building fractions from unit fractions is used to develop the ideas of fractions greater than 1 and mixed numbers

$$\boxed{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}+\frac{1}{4}=1\frac{1}{4}$$

$$\frac{5}{4}=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$$



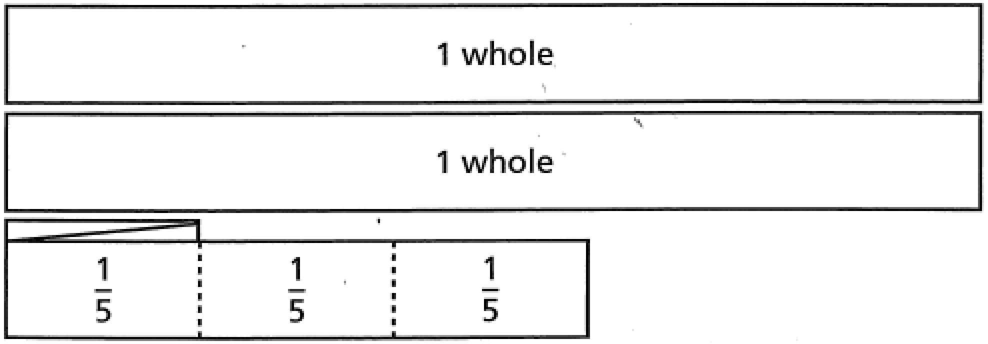
FRACTIONS GREATER THAN 1 AND MIXED NUMBERS

- Build Mixed Numbers from Unit Fractions

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

BUILD (IMPROPER??) FRACTIONS…

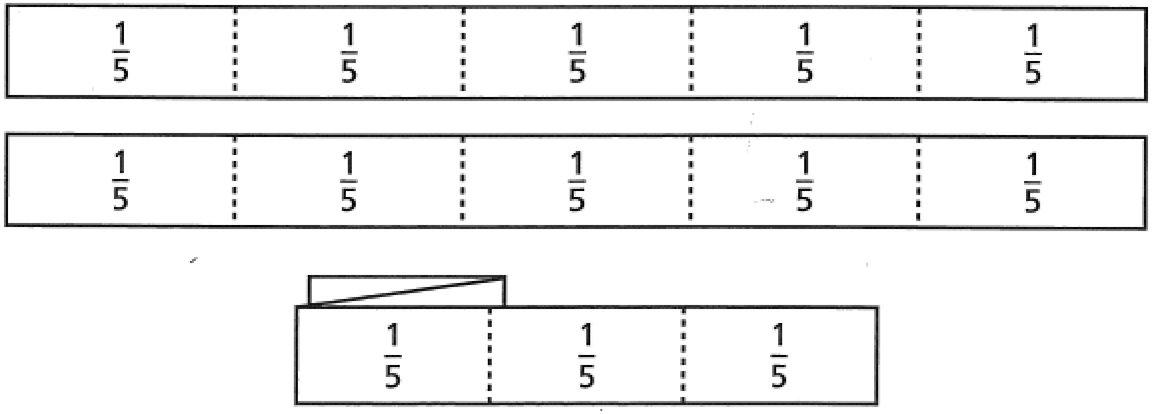
- Build 2 $\frac{3}{5}$ with your fraction strips.



- How many fifths do you have in all?
  - *5 fifths + 5 fifths + 3 fifths = 13 fifths*
- How could you write this as an improper fraction?
  - $\frac{13}{5}$

BUILD FRACTIONS…

- Check your answer by turning over the two whole fraction strips.



MIXED NUMBERS AND…

- For the mixed number $4\,\frac{2}{5}$, how do you know how many *1 whole strips* are needed to make the mixed number?
  - *Look at the whole number…4*
- How do you know how many more fraction strips are needed to make the fraction in the mixed number?
  - *Look at the numerator for how many and the denominator for what type…*$\frac{2}{5}$

MIXED NUMBERS AND FRACTIONS

- Use your fraction strips to rewrite 4 2/5 into a fraction.
- Record and discuss what you did?

Whole numbers plus the fractions

$$4\ \frac{2}{5} = 1 + 1 + 1 + 1 + \frac{2}{5}$$

Write the number of parts to each whole

$$4\ \frac{2}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{2}{5}$$

Write the total number of parts

$$4\ \frac{2}{5} = \frac{22}{5}$$

FRACTIONS

- For the fraction $\frac{19}{5}$, how do you know what type of fraction strip to use to build this fraction?
  - *Look at the denominator…fifths*
- How do you know how many fifths are needed to make the fraction with fraction strips?
  - *Look at the numerator…19*

FRACTIONS AND MIXED NUMBERS

- Students will use their fraction strips to rewrite $\frac{19}{5}$ into a mixed number with the following sequence.

Number of parts to each whole

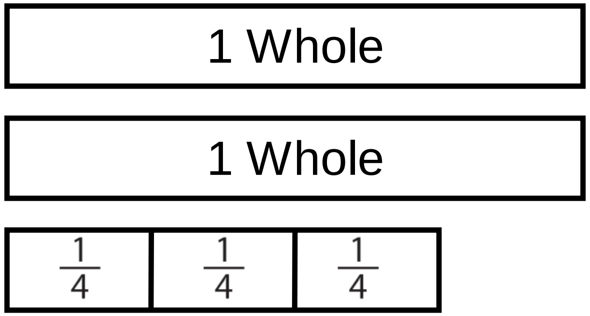
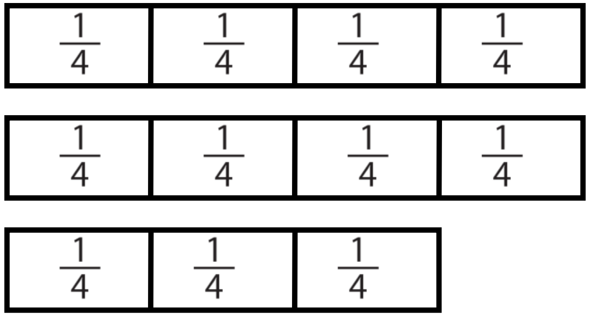
$$\frac{19}{5} = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{4}{5}$$

Whole number plus the fraction

$$\frac{19}{5} = 1 + 1 + 1 + \frac{4}{5} \text{ Total}$$

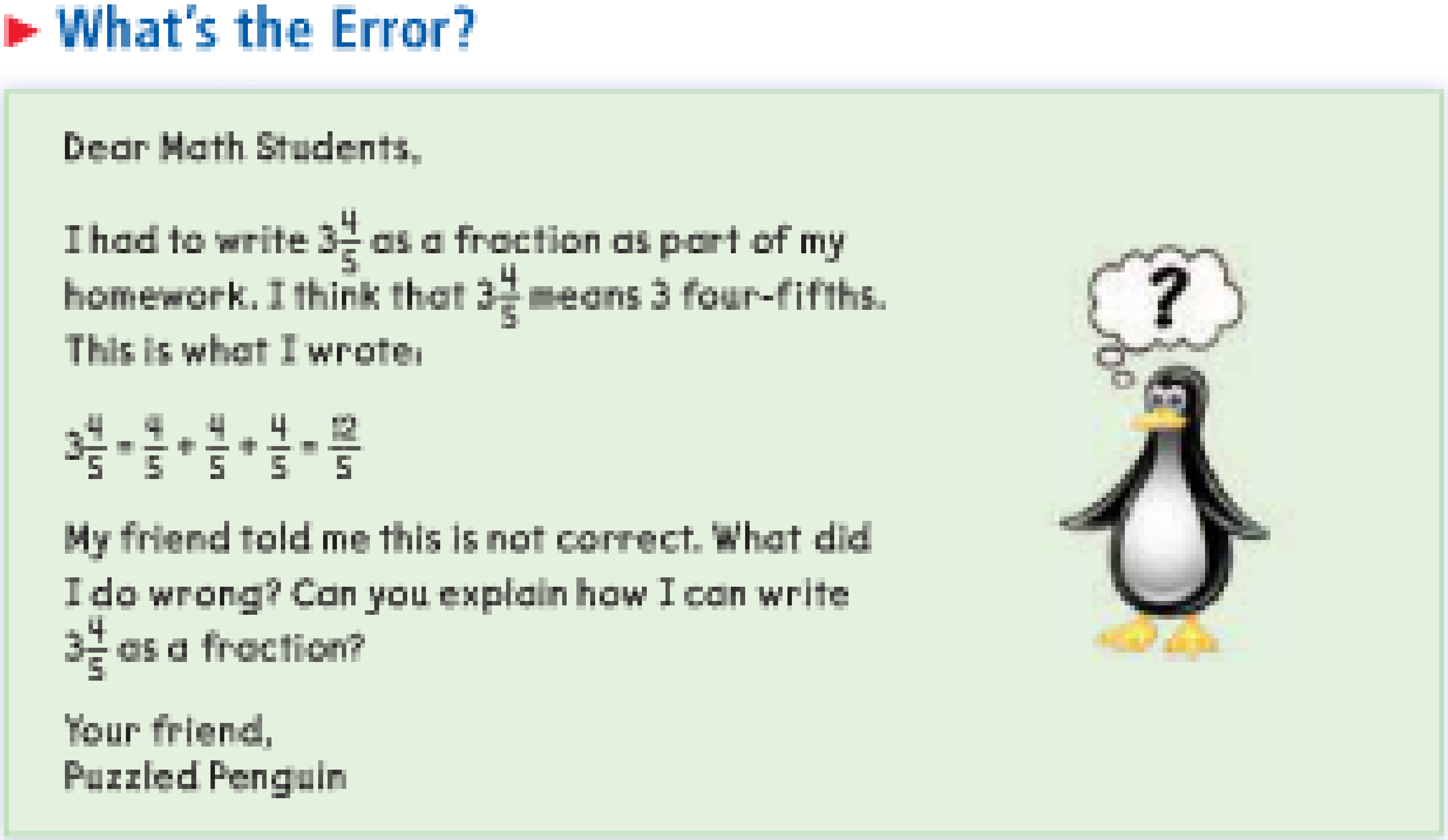
$$\frac{19}{5} = 3\ \frac{4}{5}$$

FRACTION G.O. SHEETS

Mixed Number	**Mixed Number Drawing**
$2\frac{3}{4}$	
Fraction	**Fraction Drawing**
$\frac{11}{4}$ *“4 times 2 + 3”*	

MATH TALK

- What’s the Error?



What’s the Error?

Dear Math Students,

I had to write $3\frac{4}{5}$ as a fraction as part of my homework. I think that $3\frac{4}{5}$ means 3 four-fifths. This is what I wrote:

$$3\frac{4}{5} = \frac{4}{5} + \frac{4}{5} + \frac{4}{5} = \frac{12}{5}$$

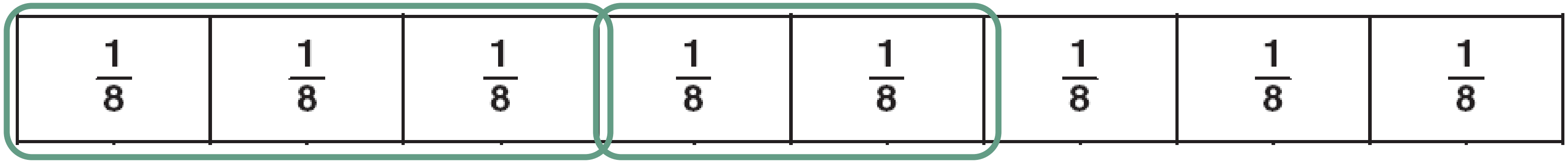
My friend told me this is not correct. What did I do wrong? Can you explain how I can write $3\frac{4}{5}$ as a fraction?

Your friend,
Puzzled Penguin

YOU TRY! ADD AND SUBTRACT FRACTIONS

- Use fraction strips to add $\frac{3}{8} + \frac{2}{8}$

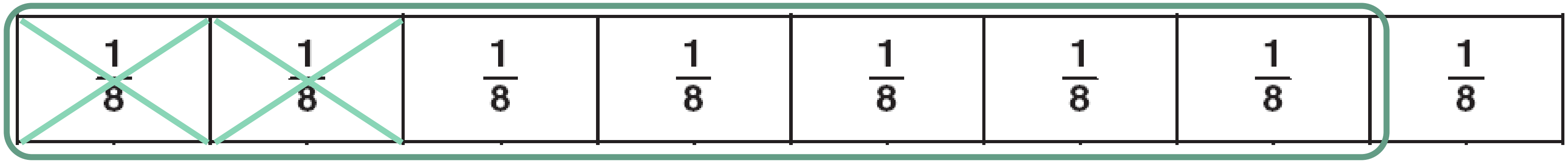
$$\frac{3}{8} \quad + \quad \frac{2}{8} \quad = \quad \frac{5}{8}$$



$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

- Use fraction strips to subtract $\frac{7}{8} - \frac{2}{8}$

$$\frac{7}{8} \quad - \quad \frac{2}{8} \quad = \quad \frac{5}{8}$$

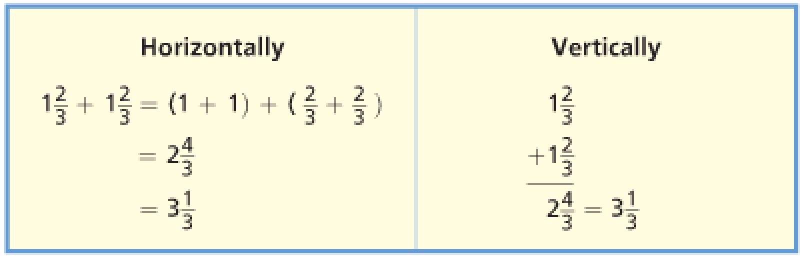


$\cancel{\frac{1}{8}}$	$\cancel{\frac{1}{8}}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

ADD LIKE MIXED NUMBERS

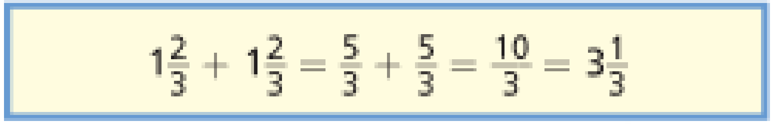
Add

- Add whole number parts and fractions parts separately and regroup if needed.



Horizontally	Vertically
$1\frac{2}{3} + 1\frac{2}{3} = (1 + 1) + (\frac{2}{3} + \frac{2}{3})$ $= 2\frac{4}{3}$ $= 3\frac{1}{3}$	$\begin{array}{r} 1\frac{2}{3} \\ +1\frac{2}{3} \\ \hline 2\frac{4}{3} = 3\frac{1}{3} \end{array}$

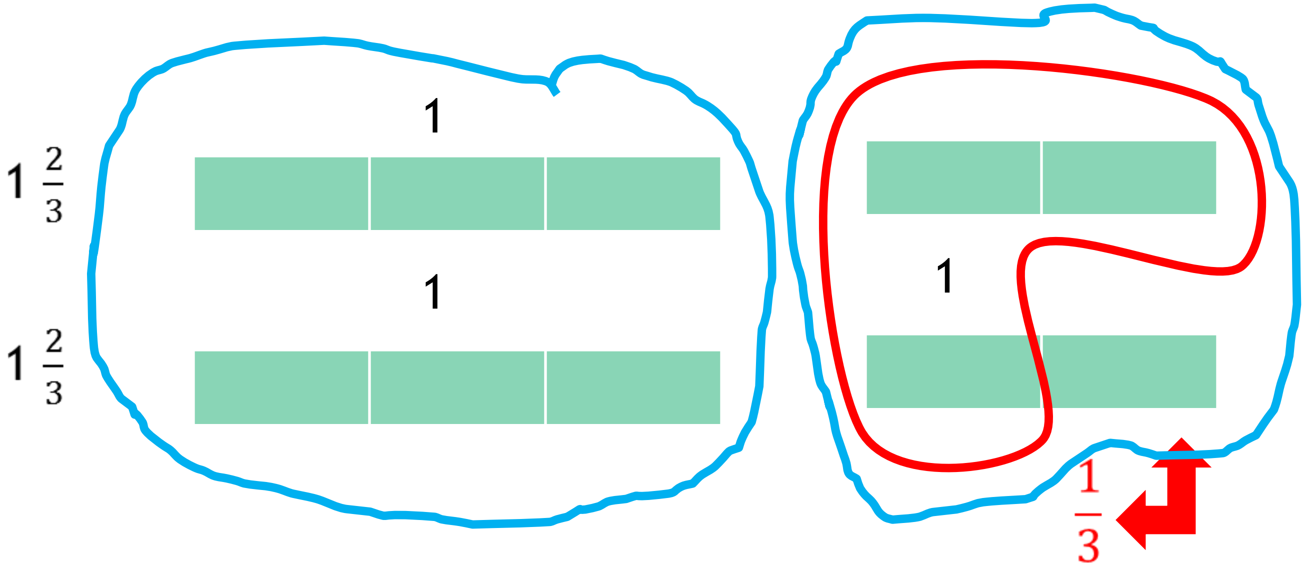
- Rewrite the mixed numbers as fractions and add



$$1\frac{2}{3} + 1\frac{2}{3} = \frac{5}{3} + \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$$

ADD LIKE MIXED NUMBERS

- Draw a picture to add and regroup.



$$1\frac{2}{3} + 1\frac{2}{3} = \quad 1 + 1 + \frac{2}{3} + \frac{2}{3} = \quad 1 + 1 + 1 + \frac{1}{3} = \quad 3\frac{1}{3}$$

$$1\frac{2}{3} + 1\frac{2}{3} = \quad 2 + \frac{4}{3} = \quad 2 + 1 + \frac{1}{3} = \quad 3\frac{1}{3}$$

- Which way did you see?

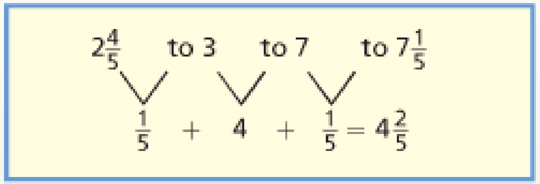
SUBTRACT LIKE MIXED NUMBERS

Subtract

- Subtract whole number parts and fraction parts separately, ungroup first

$$\begin{array}{r} 6\frac{6}{5} \\ \cancel{7}\frac{\cancel{1}}{5} \\ -\ 2\frac{4}{5} \\ \hline 4\frac{2}{5} \end{array}$$

- Add on from the lesser number to the greater number.



- Rewrite the mixed numbers as fractions and subtract.

$$7\frac{1}{5} - 2\frac{4}{5} = \frac{36}{5} - \frac{14}{5} = \frac{22}{5} = 4\frac{2}{5}$$

SUBTRACT MIXED NUMBERS WITH RENAMING

- Rename mixed numbers before you can subtract

$7\frac{1}{5} - 2\frac{4}{5} =$

Since $\frac{1}{5}$ is less than $\frac{4}{5}$ = , rename $7\frac{1}{5} = 6 + \frac{5}{5} + \frac{1}{5} = 6\frac{6}{5}$

What is the difference between the fractions?

$$\begin{array}{rcr} 7\frac{1}{5} & = & 6\frac{6}{5} \\ -\ 2\frac{4}{5} & = & -\ 2\frac{4}{5} \\ \hline & & \frac{2}{5} \end{array}$$

What is the difference between the whole numbers?

$$\begin{array}{rcr} 7\frac{1}{5} & = & 6\frac{6}{5} \\ -\ 2\frac{4}{5} & = & -\ 2\frac{4}{5} \\ \hline & & 4\frac{2}{5} \end{array}$$

MATH TALK

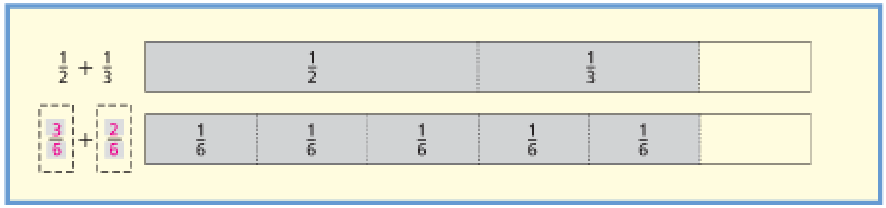
- What is the Error – or Anticipate why kids do not understand this…

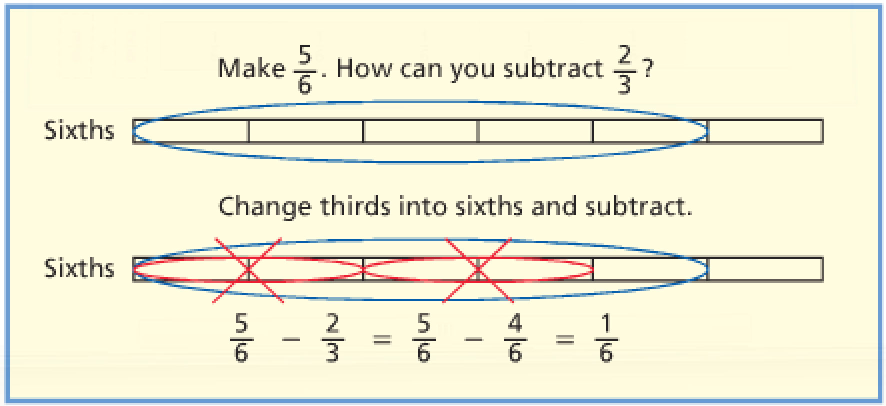
$$\begin{array}{rcr} 5\,\frac{2}{5} & = & 5\,\frac{7}{5} \\ -\ 3\,\frac{4}{5} & = & -\ 3\,\frac{4}{5} \\ \hline & & 2\,\frac{3}{5} \end{array}$$

- To get $\frac{7}{5}$ in the top number you have to ungroup one of the wholes in 5 to get $\frac{5}{5}$. This leaves you with one less whole than you had before, so you should have changed 5 to 4. This would give you $4\,\frac{7}{5}$ – $3\,\frac{4}{5}$ which is $1\,\frac{3}{5}$
- Do they understand 5/5 = 1?
  - Use fraction strips as a mathematical tool

ADD AND SUBTRACT UNLIKE FRACTIONS AND MIXED NUMBERS

- Math drawings!!
- Math Talk!

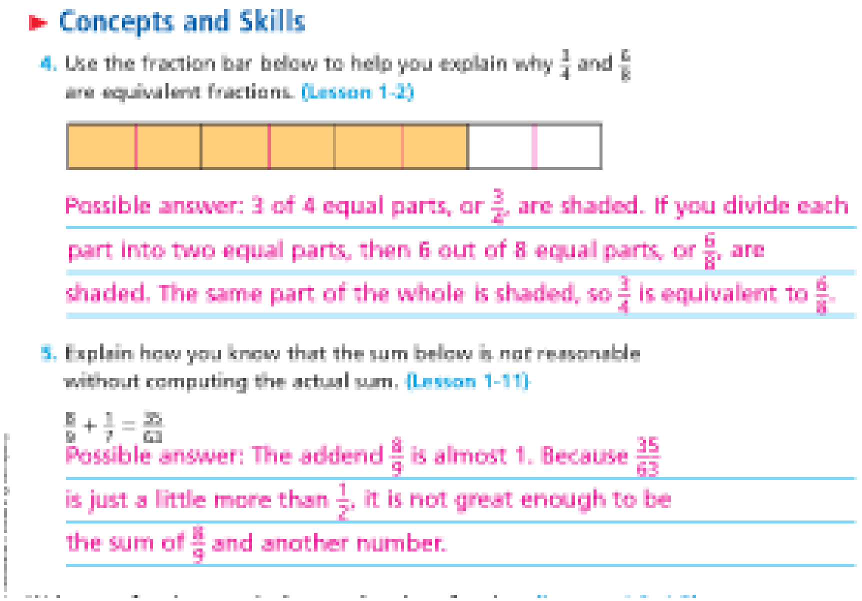




Make $\frac{5}{6}$. How can you subtract $\frac{2}{3}$?

Change thirds into sixths and subtract.

$$\frac{5}{6} - \frac{2}{3} = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$$



Concepts and Skills

4. Use the fraction bar below to help you explain why $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions. (Lesson 1-2)

Possible answer: 3 of 4 equal parts, or $\frac{3}{4}$, are shaded. If you divide each part into two equal parts, then 6 out of 8 equal parts, or $\frac{6}{8}$, are shaded. The same part of the whole is shaded, so $\frac{3}{4}$ is equivalent to $\frac{6}{8}$.

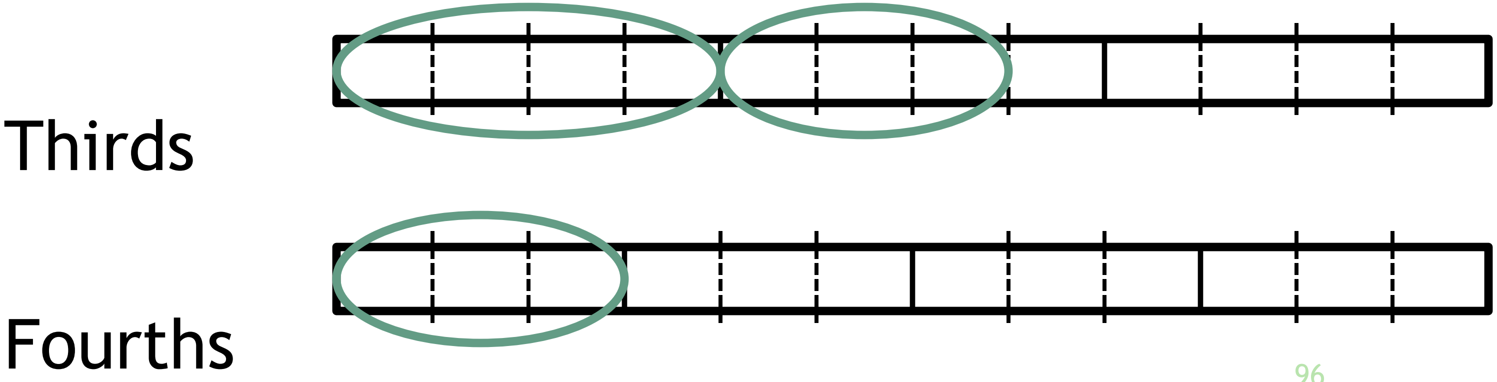
5. Explain how you know that the sum below is not reasonable without computing the actual sum. (Lesson 1-11)

$\frac{8}{9} + \frac{1}{7} = \frac{35}{63}$

Possible answer: The addend $\frac{8}{9}$ is almost 1. Because $\frac{35}{63}$ is just a little more than $\frac{1}{2}$, it is not great enough to be the sum of $\frac{8}{9}$ and another number.

RENAME FRACTIONS TO ADD

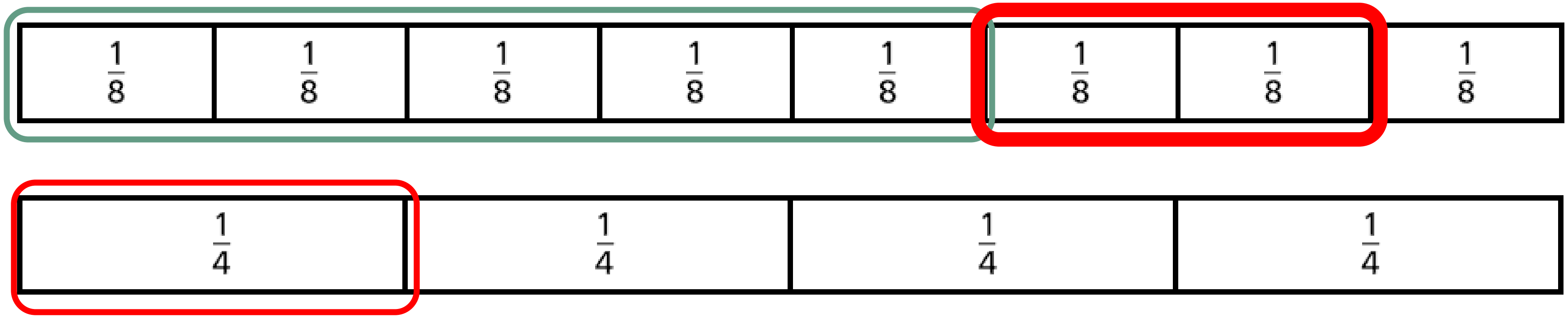
- Why can’t you add $\frac{1}{3} + \frac{1}{4}$ easily?
  - You can’t tell where $\frac{1}{4}$ is on the $\frac{1}{3}$ bar.
- How can you divide both fourths and thirds into the same sized unit fractions?
  - Split each third into 4 parts and split each fourth into 3 parts. That gives us twelfths on both bars.
- How many twelfths is $\frac{1}{3}$ and $\frac{1}{4}$?
- What is the total? $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$



STRATEGIES FOR RENAMING

- Use strategies for comparing fractions to rewrite fractions for adding. (MP repeated reasoning)
- Connect symbols and models. (MP reason abstract/quantitatively)
- Add $\frac{5}{8} + \frac{1}{4}$

$$\frac{5}{8} + \frac{1}{4} = \frac{5}{8} + \frac{}{8} = \frac{5+2}{8} = \frac{7}{8}$$



1/8 +1/4 Doesn’t tell us what the total is called, so how can we decide?

Divide the bar and rename a unit fraction that can also make 1/8 and 1/4

Find a Common Denominator, 1/4 = 2/8

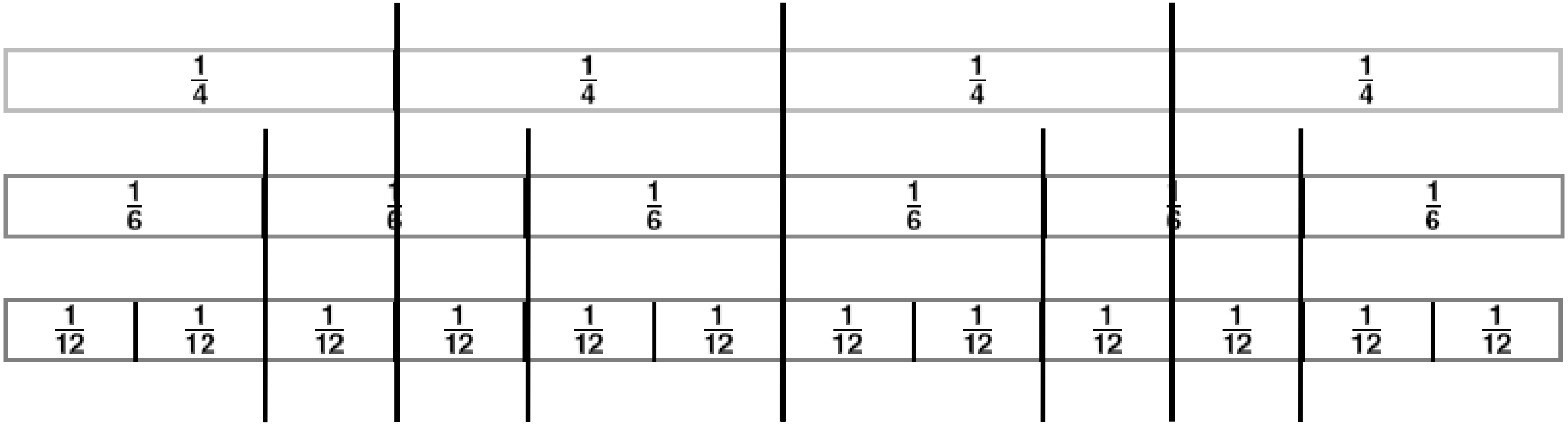
USING FRACTION STRIPS

- Please take out your fraction strip template and follow along with the next example.

FRACTION STRIPS TO ADD

- How to use fraction strips to add 3/4 + 5/6

$$\frac{3}{4} + \frac{5}{6} = \frac{\quad}{12} + \frac{\quad}{12} = \frac{9+10}{12} = \frac{19}{12} = 1\,\frac{7}{12}$$



RENAME THE MIXED NUMBERS AND SUBTRACT

- Subtract $5\frac{1}{2} - 1\frac{2}{3}$

$$\frac{1\ x\ 3}{2\ x\ 3} = \frac{3}{6} \qquad \frac{2\ x\ 2}{3\ x\ 2} = \frac{4}{6}$$

- Find the LCD
  - Multiply the 2 denominators: 2 x 3

- Rename the mixed numbers.

$$5\frac{1}{2} = 5\frac{3}{6} = 4\frac{9}{6}$$

- Find the difference.

$$-1\frac{2}{3} = 1\frac{4}{6} = -1\frac{4}{6}$$

$$3\frac{5}{6}$$

MATH TALK

- What is the Error? $$\frac{3}{5}+\frac{3}{10}=\frac{6}{15}$$

- Describe 2 ways to justify why this is wrong?

- The fraction 3/5 is more than ½, but the sum is less than ½. Therefore it cannot be correct.

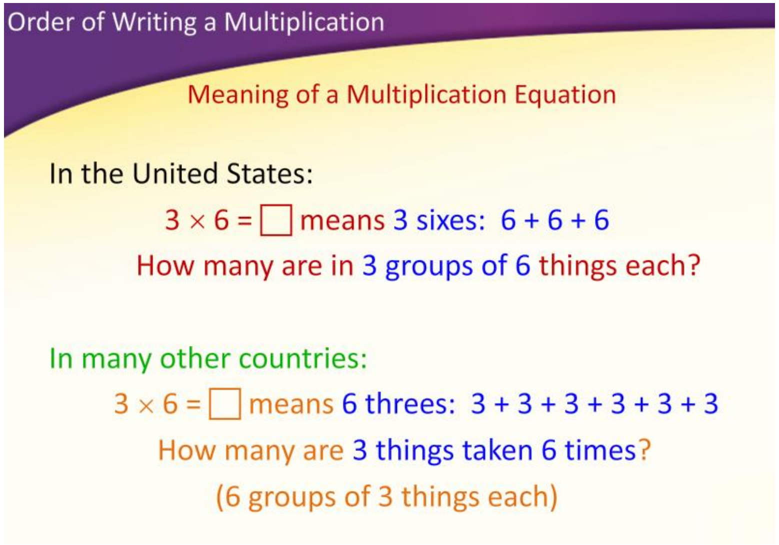
- To add two fractions, they have to have the same denominator. That is, they have to be made from the same unit fractions. The fraction 2/5 is made from fifths, while the fraction 3/10 is made from tenths. Because these unit fractions are different sizes, the fractions cannot be combined.

- If we change the fraction 3/5 to equivalent fraction 6/10, then the two addends will have the denominator 10. That is both will be made from the same unit fraction 1/10. Then we can add them.

- When we find 6/10 + 3/10, we are adding 6 tenths and 3 tenths, which is 9 tenths, or 9/10. So we add the numerators and leave the denominator as tenths.

UNDERSTANDING EXPRESSIONS

- How would you explain the following expressions?
  - 5 x 2
  - 2 x 5
- Now, how would you explain the following expressions?
  - 5 x ½
  - ½ x 5



Order of Writing a Multiplication

Meaning of a Multiplication Equation

In the United States:

3 × 6 = ☐ means 3 sixes: 6 + 6 + 6

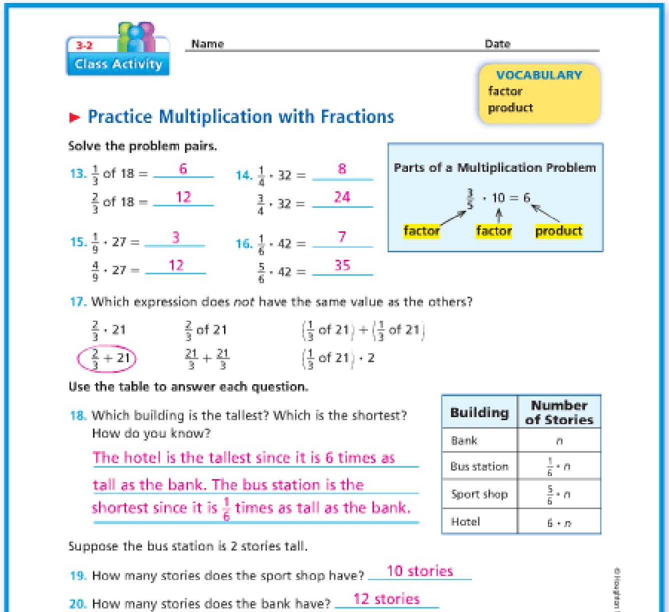
How many are in 3 groups of 6 things each?

In many other countries:

3 × 6 = ☐ means 6 threes: 3 + 3 + 3 + 3 + 3 + 3

How many are 3 things taken 6 times?

(6 groups of 3 things each)



3-2 Class Activity

Name Date

VOCABULARY
factor
product

Practice Multiplication with Fractions

Solve the problem pairs.

13. $\frac{1}{3}$ of 18 = 6
$\frac{2}{3}$ of 18 = 12

14. $\frac{1}{4} \cdot 32 =$ 8
$\frac{3}{4} \cdot 32 =$ 24

15. $\frac{1}{9} \cdot 27 =$ 3
$\frac{4}{9} \cdot 27 =$ 12

16. $\frac{1}{6} \cdot 42 =$ 7
$\frac{5}{6} \cdot 42 =$ 35

Parts of a Multiplication Problem

$\frac{3}{5} \cdot 10 = 6$

factor factor product

17. Which expression does *not* have the same value as the others?

$\frac{2}{3} \cdot 21$ $\frac{2}{3}$ of 21 $\left(\frac{1}{3} \text{ of } 21\right) + \left(\frac{1}{3} \text{ of } 21\right)$

$\frac{2}{3} + 21$ $\frac{21}{3} + \frac{21}{3}$ $\left(\frac{1}{3} \text{ of } 21\right) \cdot 2$

Use the table to answer each question.

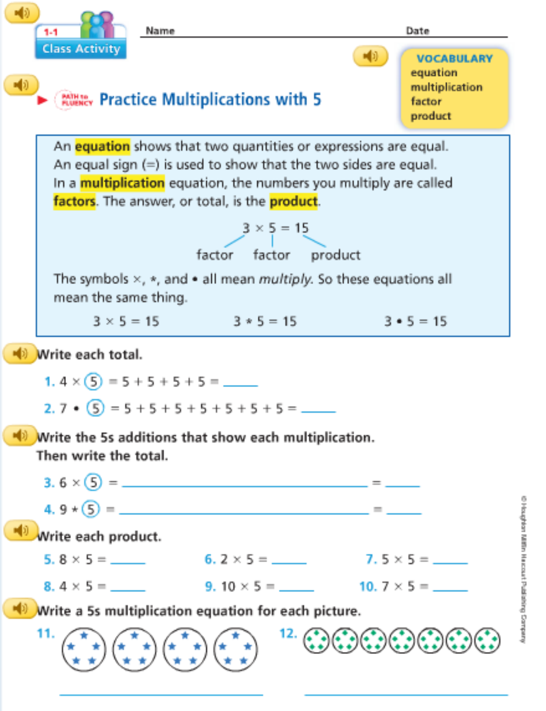
Building	Number of Stories
Bank	n
Bus station	$\frac{1}{6} \cdot n$
Sport shop	$\frac{5}{6} \cdot n$
Hotel	$6 \cdot n$

18. Which building is the tallest? Which is the shortest? How do you know?
The hotel is the tallest since it is 6 times as tall as the bank. The bus station is the shortest since it is $\frac{1}{6}$ times as tall as the bank.

Suppose the bus station is 2 stories tall.

19. How many stories does the sport shop have? 10 stories

20. How many stories does the bank have? 12 stories



1-1 Class Activity

Name Date

VOCABULARY
equation
multiplication
factor
product

Practice Multiplications with 5

An **equation** shows that two quantities or expressions are equal. An equal sign (=) is used to show that the two sides are equal. In a **multiplication** equation, the numbers you multiply are called **factors**. The answer, or total, is the **product**.

3 × 5 = 15

factor factor product

The symbols ×, *, and • all mean *multiply*. So these equations all mean the same thing.

3 × 5 = 15 3 * 5 = 15 3 • 5 = 15

Write each total.

1. 4 × ⑤ = 5 + 5 + 5 + 5 = ____
2. 7 • ⑤ = 5 + 5 + 5 + 5 + 5 + 5 + 5 = ____

Write the 5s additions that show each multiplication. Then write the total.

3. 6 × ⑤ = ____________ = ____
4. 9 * ⑤ = ____________ = ____

Write each product.

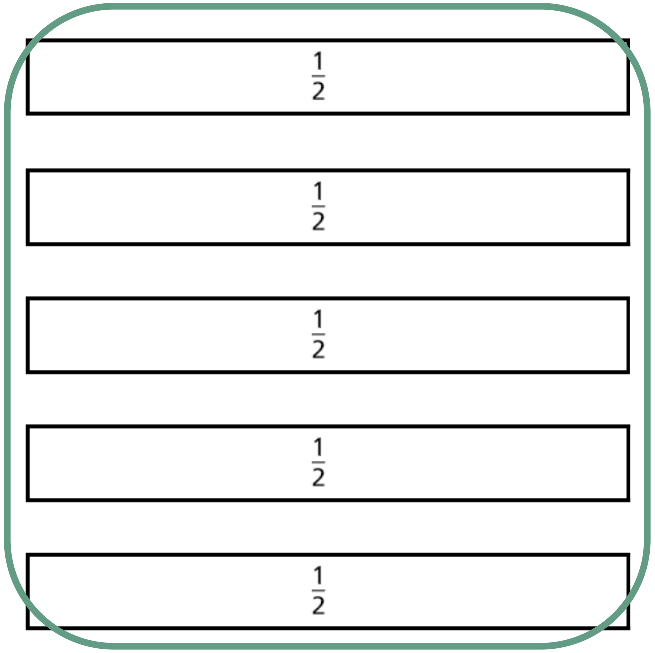
5. 8 × 5 = ____ 6. 2 × 5 = ____ 7. 5 × 5 = ____
8. 4 × 5 = ____ 9. 10 × 5 = ____ 10. 7 × 5 = ____

Write a 5s multiplication equation for each picture.

11. ____________ 12. ____________

5 X ½

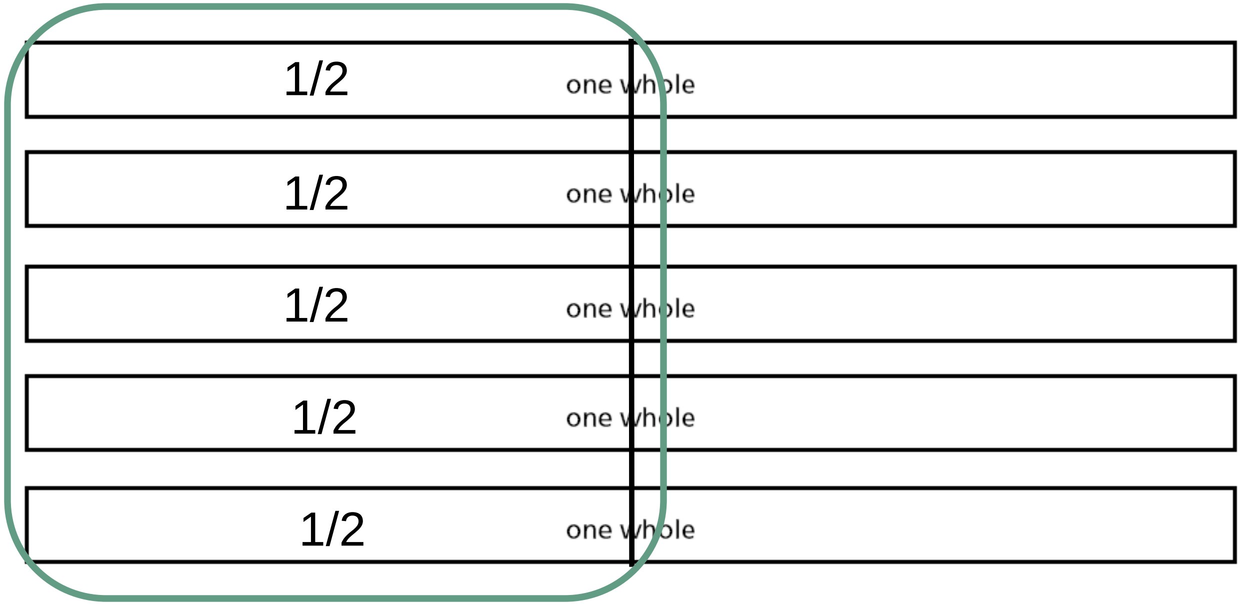
- "5 groups of ½"
  - We need to draw 5 "halves" using fraction strips.



1/2 + 1/2 + 1/2 + 1/2 + 1/2 = 5/2 = 2 1/2

½ X 5

- “1/2 of 5 wholes”
  - We need to draw 5 “whole” fraction strips.



1/2 one whole

1/2 one whole

1/2 one whole

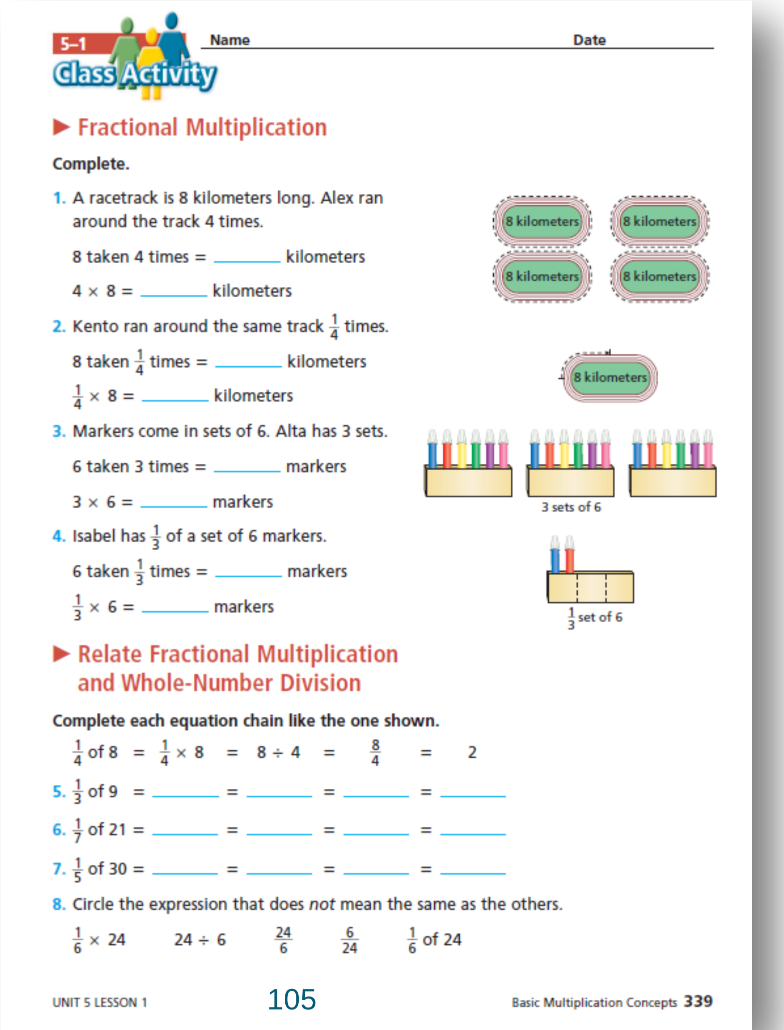
1/2 one whole

1/2 one whole

1/2 + 1/2 + 1/2 + 1/2 + 1/2 = 5/2 = 2 1/2

MULTIPLY A WHOLE NUMBER BY A FRACTION

- Begin by multiplying a whole number by a fraction
  - Grade 4, multiplication of a fraction by a whole number as repeated addition
  - $3 \cdot \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$
  - Grade 5 multiplication of a whole number by a fraction as finding that fraction of the whole number



5–1 Class Activity

Name ______ Date ______

▶ Fractional Multiplication

Complete.

1. A racetrack is 8 kilometers long. Alex ran around the track 4 times.

8 taken 4 times = ______ kilometers

4 × 8 = ______ kilometers

2. Kento ran around the same track $\frac{1}{4}$ times.

8 taken $\frac{1}{4}$ times = ______ kilometers

$\frac{1}{4} \times 8 =$ ______ kilometers

3. Markers come in sets of 6. Alta has 3 sets.

6 taken 3 times = ______ markers

3 × 6 = ______ markers

4. Isabel has $\frac{1}{3}$ of a set of 6 markers.

6 taken $\frac{1}{3}$ times = ______ markers

$\frac{1}{3} \times 6 =$ ______ markers

▶ Relate Fractional Multiplication and Whole-Number Division

Complete each equation chain like the one shown.

$\frac{1}{4}$ of 8 = $\frac{1}{4} \times 8$ = $8 \div 4$ = $\frac{8}{4}$ = 2

5. $\frac{1}{3}$ of 9 = ______ = ______ = ______ = ______

6. $\frac{1}{7}$ of 21 = ______ = ______ = ______ = ______

7. $\frac{1}{5}$ of 30 = ______ = ______ = ______ = ______

8. Circle the expression that does *not* mean the same as the others.

$\frac{1}{6} \times 24$ $24 \div 6$ $\frac{24}{6}$ $\frac{6}{24}$ $\frac{1}{6}$ of 24

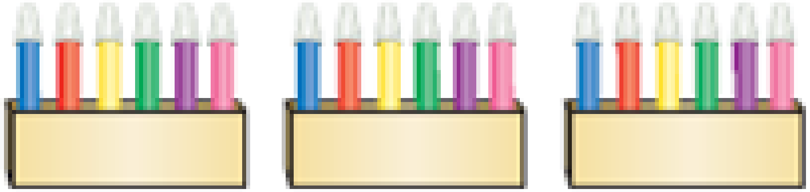
UNIT 5 LESSON 1 Basic Multiplication Concepts **339**

MULTIPLYING BY A UNIT FRACTION

3. Markers come in sets of 6. Alta has 3 sets.

6 taken 3 times = ________ markers

$3 \times 6 =$ ________ markers

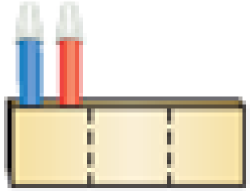


3 sets of 6

4. Isabel has $\frac{1}{3}$ of a set of 6 markers.

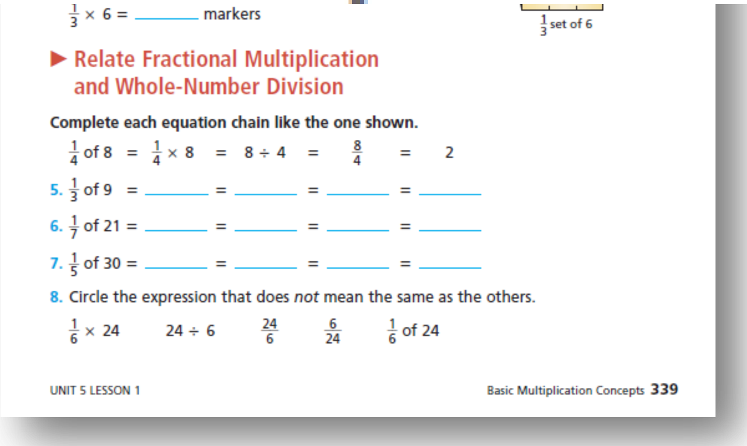
6 taken $\frac{1}{3}$ times = ________ markers

$\frac{1}{3} \times 6 =$ ________ markers



$\frac{1}{3}$ set of 6

- $\frac{1}{3}$ *is 1 of 3 equal parts*
- $\frac{1}{3} \cdot 6$ *or* $\frac{1}{3}$ *of* 6
  - Requires dividing 6 into 3 equal parts



$\frac{1}{3} \times 6 =$ ________ markers

$\frac{1}{3}$ set of 6

▶ Relate Fractional Multiplication and Whole-Number Division

Complete each equation chain like the one shown.

$\frac{1}{4}$ of 8 = $\frac{1}{4} \times 8$ = $8 \div 4$ = $\frac{8}{4}$ = 2

5. $\frac{1}{3}$ of 9 = ________ = ________ = ________ = ________

6. $\frac{1}{7}$ of 21 = ________ = ________ = ________ = ________

7. $\frac{1}{5}$ of 30 = ________ = ________ = ________ = ________

8. Circle the expression that does *not* mean the same as the others.

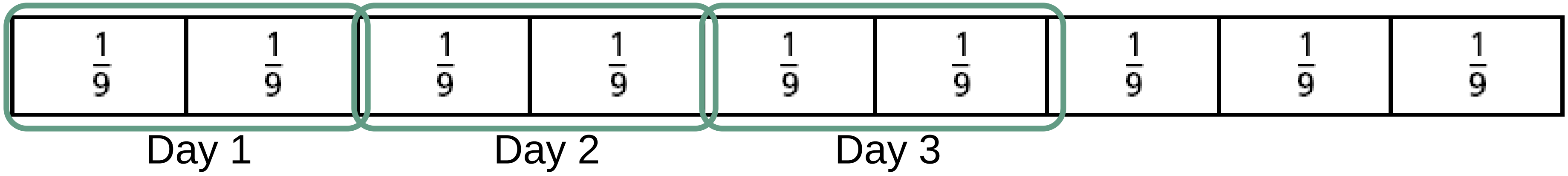
$\frac{1}{6} \times 24$ $24 \div 6$ $\frac{24}{6}$ $\frac{6}{24}$ $\frac{1}{6}$ of 24

UNIT 5 LESSON 1 Basic Multiplication Concepts **339**

MULTIPLYING WITH FRACTION STRIPS

- Bob eats 2/9ths of his Easter candy each day, for three days in a row. After the third day of eating, what fractional part of his Easter candy did he eat?

- Use a fraction strip to show how much he ate.



- Write an addition equation to show how much he ate.

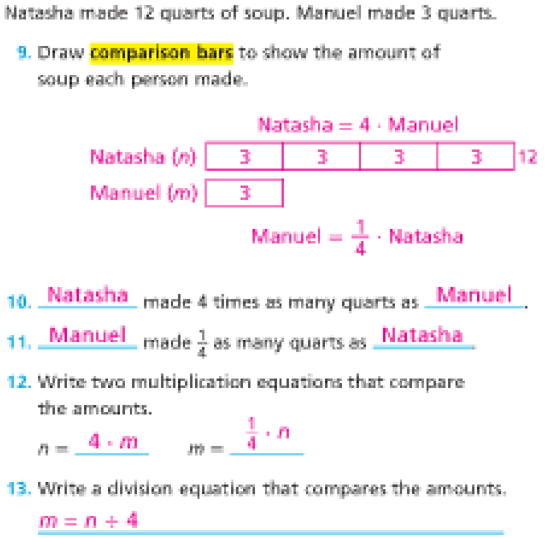
2/9 + 2/9 + 2/9 = 6/9

3 x 2/9 = 6/9

- Now, write this as a multiplication equation.

COMPARISON PROBLEMS

- Multiplying using unit fraction language
  - If a quantity *b* is *n* times a quantity *a*, then *a* is $\frac{1}{n}$ times *b*
    - 6(b) is 3(n) times 2(a)
      - $6 = 3 \cdot 2 \quad (b = n \cdot a)$
    - 2(a) is $\frac{1}{3}$ ($\frac{1}{n}$) times 6(b)
      - $2 = \frac{1}{3} \cdot 6 \quad (a = \frac{1}{n} \cdot b)$



Natasha made 12 quarts of soup. Manuel made 3 quarts.

9. Draw comparison bars to show the amount of soup each person made.

10. Natasha made 4 times as many quarts as Manuel.

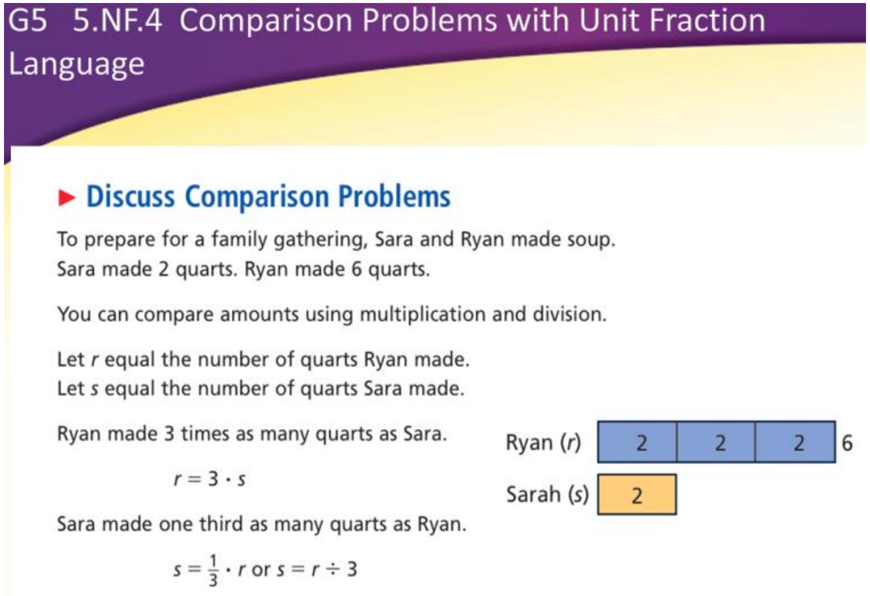
11. Manuel made $\frac{1}{4}$ as many quarts as Natasha.

12. Write two multiplication equations that compare the amounts.

$n = 4 \cdot m \qquad m = \frac{1}{4} \cdot n$

13. Write a division equation that compares the amounts.

$m = n \div 4$



G5 5.NF.4 Comparison Problems with Unit Fraction Language

▶ **Discuss Comparison Problems**

To prepare for a family gathering, Sara and Ryan made soup. Sara made 2 quarts. Ryan made 6 quarts.

You can compare amounts using multiplication and division.

Let *r* equal the number of quarts Ryan made.
Let *s* equal the number of quarts Sara made.

Ryan made 3 times as many quarts as Sara.

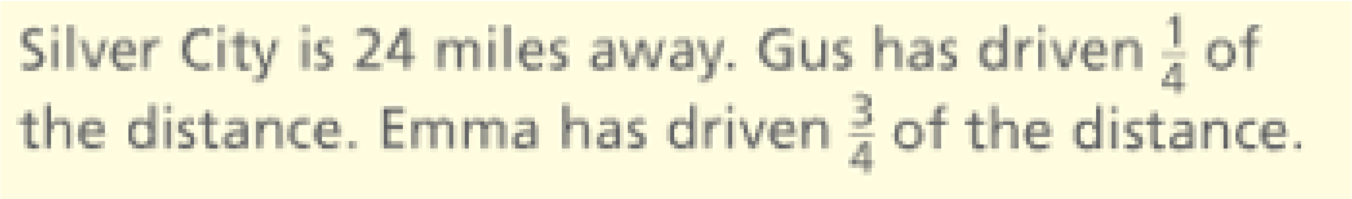
$r = 3 \cdot s$

Sara made one third as many quarts as Ryan.

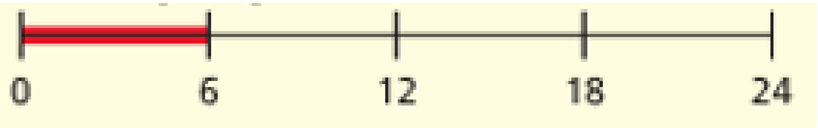
$s = \frac{1}{3} \cdot r$ or $s = r \div 3$

MULTIPLY A WHOLE NUMBER BY A NON-UNIT FRACTION

- $\frac{a}{b} \cdot \left(\frac{1}{b} \cdot n\right)$


Silver City is 24 miles away. Gus has driven $\frac{1}{4}$ of the distance. Emma has driven $\frac{3}{4}$ of the distance.

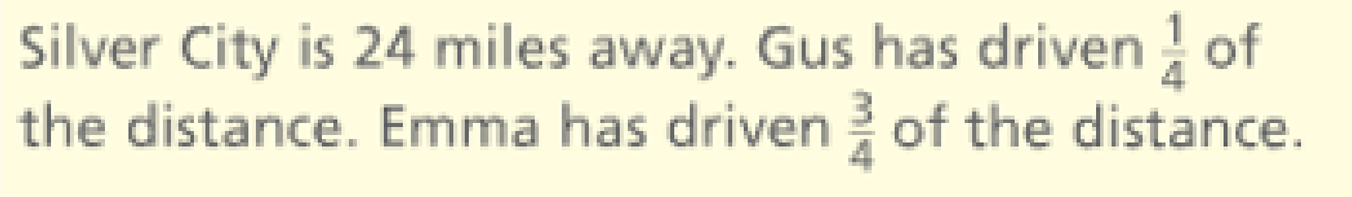
- Draw a line from 0-24. To show fourths, divide it into 4 equal parts of 6
  - $24 \div 4 = 6$
    - $\frac{1}{4}$ of the way is 6 miles
    - $\frac{2}{4}$ of the way is 12 miles
    - $\frac{3}{4}$ of the way is 18 miles



- Use the answer for 1/4 of the distance to find 3/4 of the distance.
  - ¾ is ¼ + ¼ + ¼
- ¼ of the distance is 6 miles
  - So ¾ of the distance is 6 + 6+ 6 = 18 miles

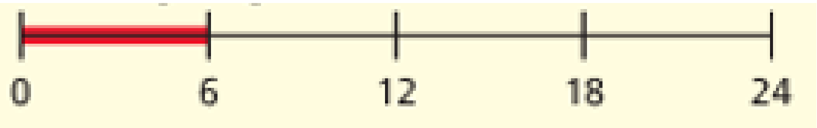
MULTIPLY A WHOLE NUMBER BY A NON-UNIT FRACTION

- $\frac{a}{b}\cdot\left(\frac{1}{b}\cdot n\right)$



Silver City is 24 miles away. Gus has driven $\frac{1}{4}$ of the distance. Emma has driven $\frac{3}{4}$ of the distance.

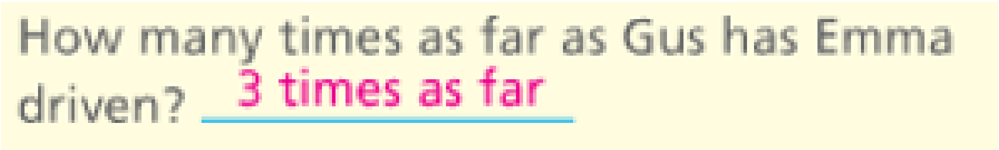
- $\frac{3}{4}\cdot 24 = 3\cdot\left(\frac{1}{4}\cdot 24\right)$
- To find $\frac{3}{4}$ of 24, calculate $\frac{1}{4}$ of 24
  - Think $\frac{1}{4}\cdot 24 = \frac{1}{4}\cdot\frac{24}{1} = \frac{24}{4} = 24 \div 4 = 6$ EMPHASIS on connection of unit fraction



How many miles has Gus driven? 6 mi

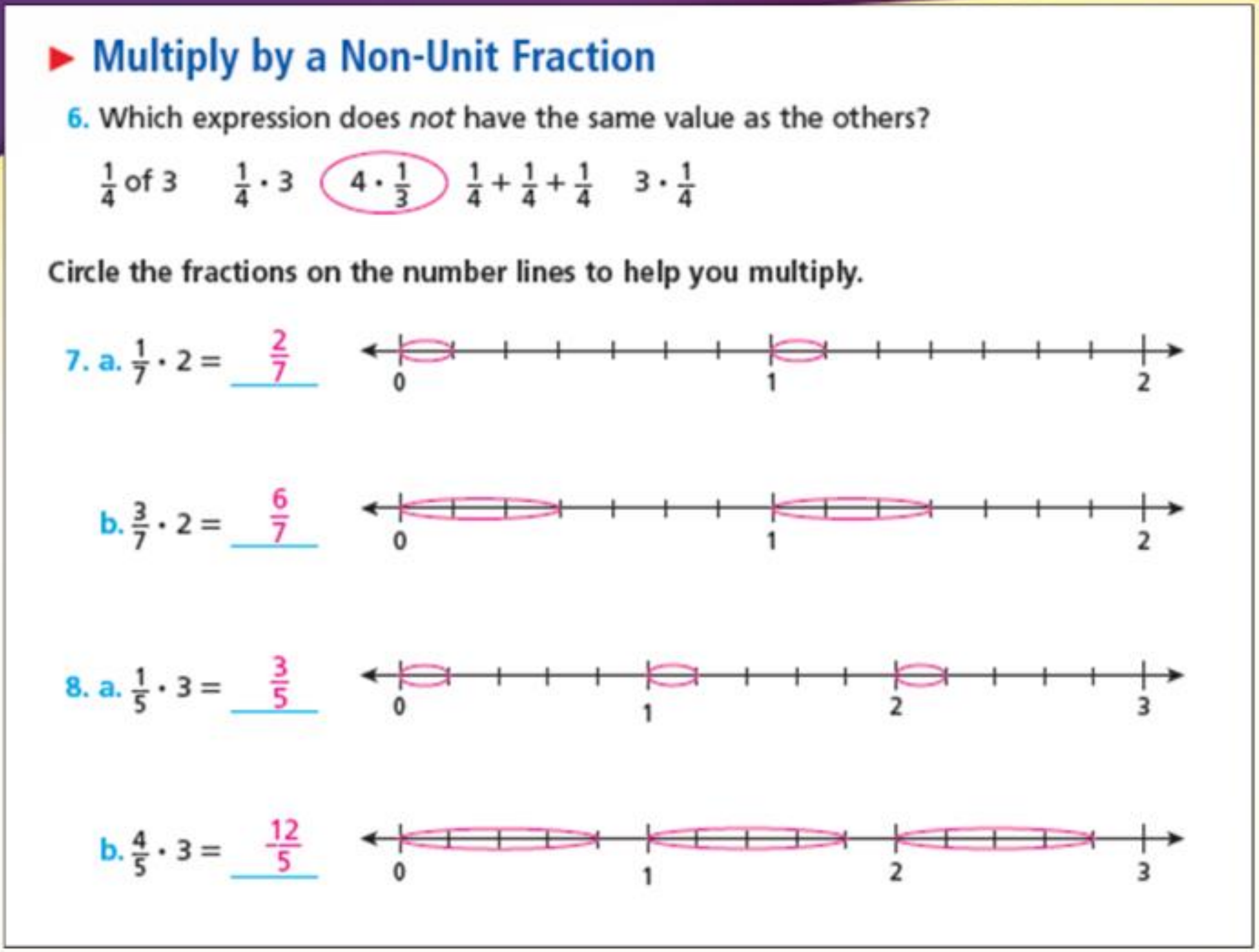
- Then multiply the result – 6 by 3

How many miles has Emma driven? 18 mi



How many times as far as Gus has Emma driven? 3 times as far

G5 5.NF.4 Any Fraction Times a Whole Number



▶ Multiply by a Non-Unit Fraction

6. Which expression does *not* have the same value as the others?

$\frac{1}{4}$ of 3 $\frac{1}{4} \cdot 3$ [✓ $4 \cdot \frac{1}{3}$] $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$ $3 \cdot \frac{1}{4}$

Circle the fractions on the number lines to help you multiply.

7. a. $\frac{1}{7} \cdot 2 =$ $\frac{2}{7}$

b. $\frac{3}{7} \cdot 2 =$ $\frac{6}{7}$

8. a. $\frac{1}{5} \cdot 3 =$ $\frac{3}{5}$

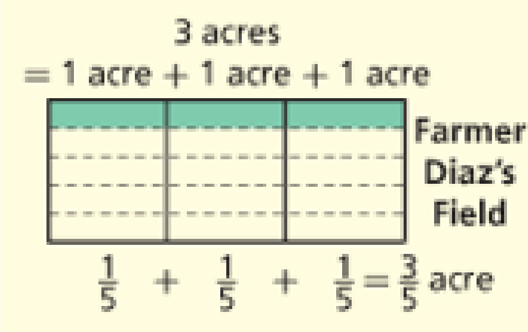
b. $\frac{4}{5} \cdot 3 =$ $\frac{12}{5}$

FRACTIONAL PRODUCTS

- Finding a unit fraction of a whole number by finding that fraction of each 1 whole and then adding the result

Farmer Diaz has 3 acres of land. He plows $\frac{1}{5}$ of this land.

- The number of acres he plows is $\frac{1}{5}$ of 3 or $\frac{1}{5} \cdot 3$
- The diagram shows Farmer Diaz's land divided vertically into 3 acres. The dashed horizontal segments divide the land into five parts.



The shaded strip is the $\frac{1}{5}$ of the land Farmer Diaz plowed.

  - The drawing shows that taking $\frac{1}{5}$ of the 3 acres is the same as taking $\frac{1}{5}$ of each acre and combining them.

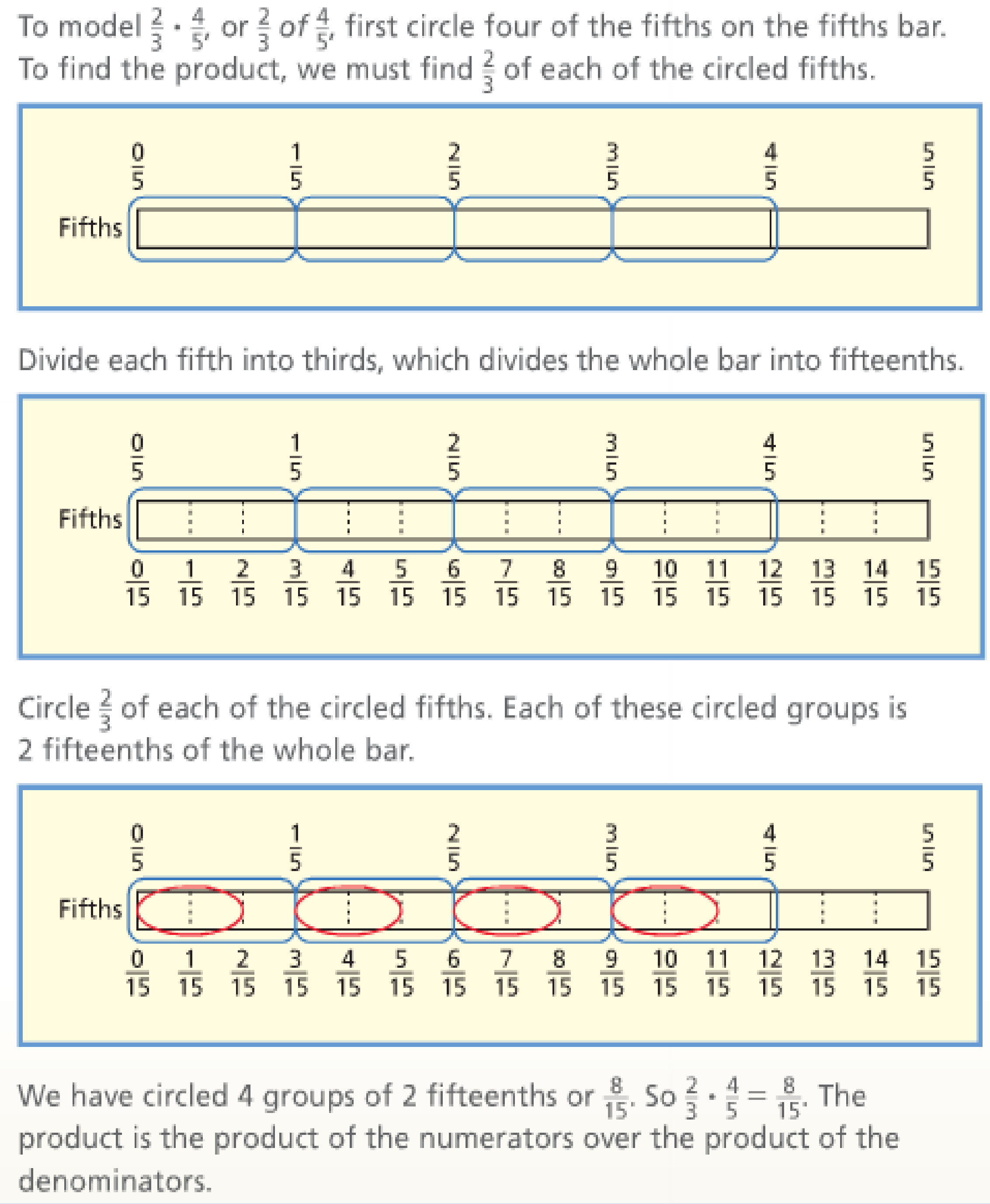
$$\begin{aligned}\frac{1}{5} \cdot 3 &= \frac{1}{5}(1 + 1 + 1)\\ &= \frac{1}{5} \cdot 1 + \frac{1}{5} \cdot 1 + \frac{1}{5} \cdot 1\\ &= \frac{1}{5} + \frac{1}{5} + \frac{1}{5}\\ &= \frac{3}{5}\end{aligned}$$

- Mathematically
  - So, $\frac{1}{5}$ of the 3 acres is $\frac{3}{5}$ acre

FRACTION BAR MODEL

- Help understand why the product of two fractions is the product of the numerators over the product of the denominators

$$\frac{2}{3}\cdot\frac{4}{5} \quad or \quad \frac{2}{3}\,of\,\frac{4}{5}$$



To model $\frac{2}{3}\cdot\frac{4}{5}$, or $\frac{2}{3}$ *of* $\frac{4}{5}$, first circle four of the fifths on the fifths bar. To find the product, we must find $\frac{2}{3}$ of each of the circled fifths.

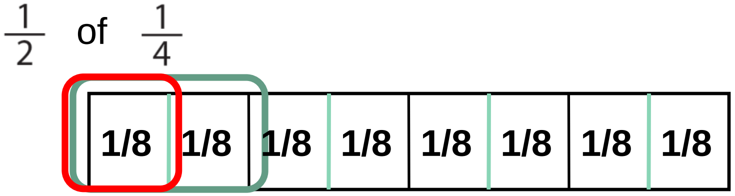
Divide each fifth into thirds, which divides the whole bar into fifteenths.

Circle $\frac{2}{3}$ of each of the circled fifths. Each of these circled groups is 2 fifteenths of the whole bar.

We have circled 4 groups of 2 fifteenths or $\frac{8}{15}$. So $\frac{2}{3}\cdot\frac{4}{5}=\frac{8}{15}$. The product is the product of the numerators over the product of the denominators.

MULTIPLY
A FRACTION BY A FRACTION

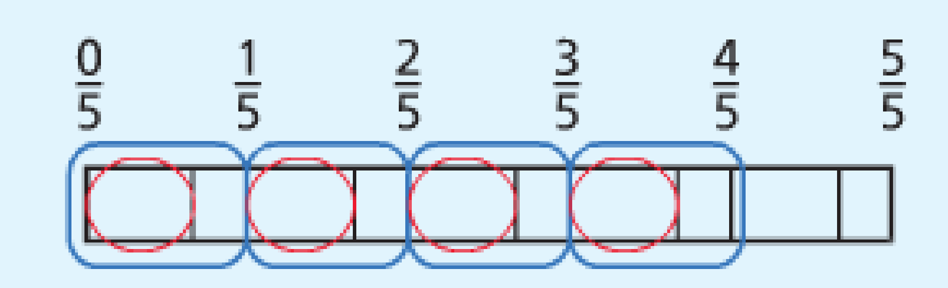
- What does $\frac{1}{2} \cdot \frac{1}{4}$ mean?
  - $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$ of the whole



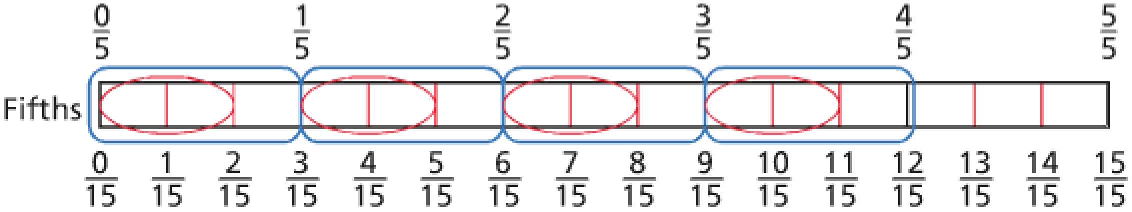
- The general formula for the product of two fractions $\frac{a}{b} x \frac{c}{d} = \frac{ac}{bd}$
  - To find the denominator: b split each $\frac{1}{d} = \ b \cdot d$
  - To find the denominator: take c groups of a of the new unit fractions $a \cdot c$
- Equation is not needed in grade 5 BUT should reason out many examples using fraction strips and number line diagrams.

WATCH FOR!

- $\frac{2}{3} of \frac{4}{5}$
  - Some students may mark only the $\frac{2}{3}$ part of each fifth and forget to mark the $\frac{1}{3}$ parts



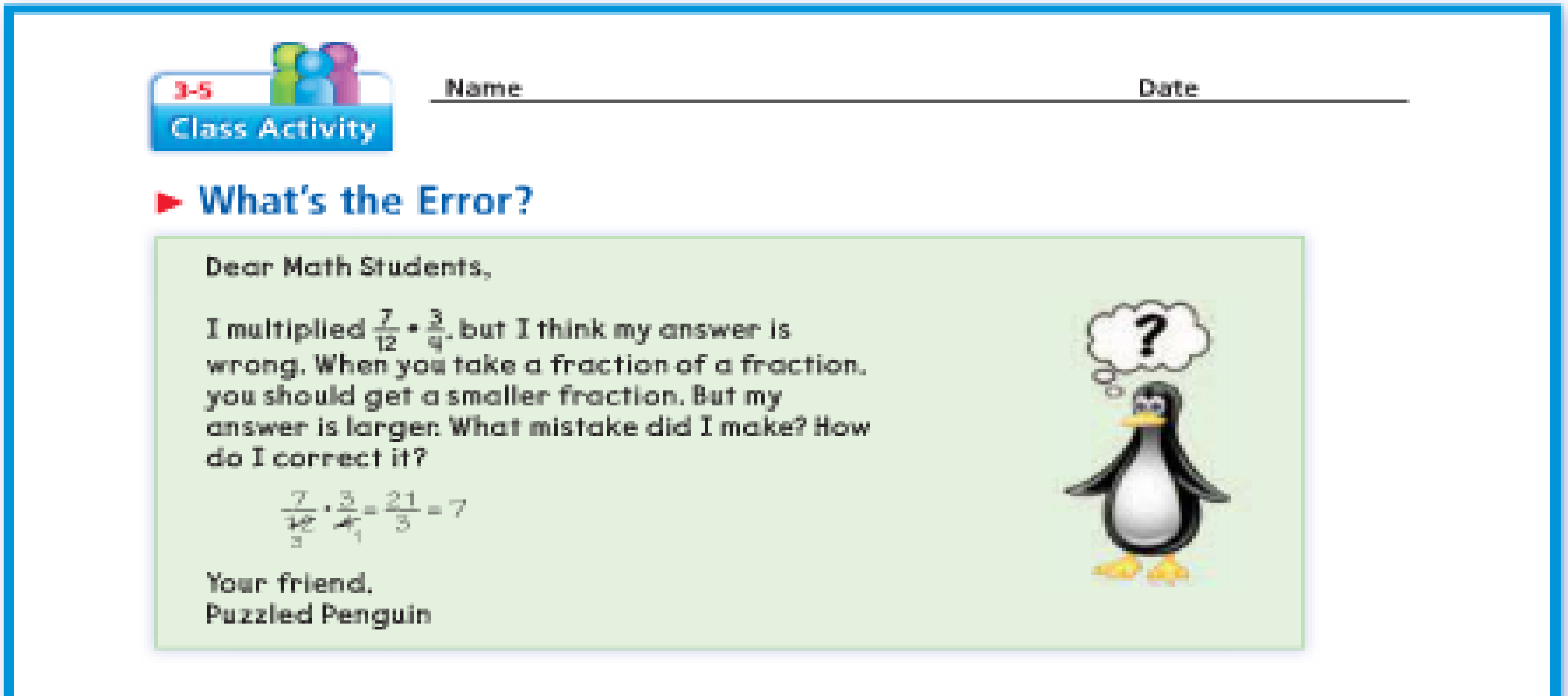
  - This would lead to an answer of $\frac{4}{10} or \frac{2}{5}$



  - Four groups of 2 fifteenths circled $\frac{2}{3}$ of $\frac{4}{5} = \frac{8}{15}$

DISCUSS

- What the error?



3-5 Class Activity

Name Date

What's the Error?

Dear Math Students,

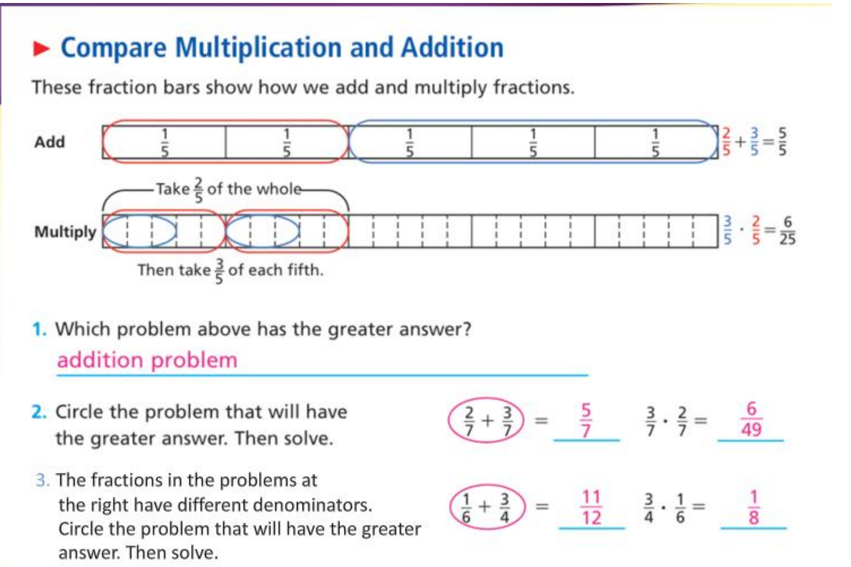
I multiplied $\frac{7}{12} \cdot \frac{3}{4}$, but I think my answer is wrong. When you take a fraction of a fraction, you should get a smaller fraction. But my answer is larger. What mistake did I make? How do I correct it?

$$\frac{7}{\underset{3}{\cancel{12}}} \cdot \frac{3}{\underset{1}{\cancel{4}}} = \frac{21}{3} = 7$$

Your friend,
Puzzled Penguin

- You divided two numbers in the denominator by the same factor. To simplify you must divide a number in the numerator and a number in the denominator by the same factor.

Compare visually to see the difference between adding and multiplying



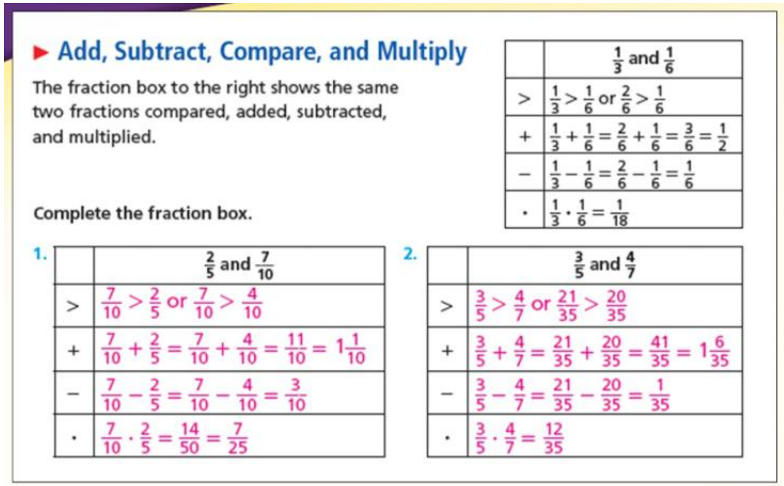
Compare Multiplication and Addition

These fraction bars show how we add and multiply fractions.

1. Which problem above has the greater answer?
addition problem

2. Circle the problem that will have the greater answer. Then solve.
$\frac{2}{7}+\frac{3}{7}=\frac{5}{7}$ $\quad \frac{3}{7}\cdot\frac{2}{7}=\frac{6}{49}$

3. The fractions in the problems at the right have different denominators. Circle the problem that will have the greater answer. Then solve.
$\frac{1}{6}+\frac{3}{4}=\frac{11}{12}$ $\quad \frac{3}{4}\cdot\frac{1}{6}=\frac{1}{8}$



Add, Subtract, Compare, and Multiply

The fraction box to the right shows the same two fractions compared, added, subtracted, and multiplied.

	$\frac{1}{3}$ and $\frac{1}{6}$
>	$\frac{1}{3}>\frac{1}{6}$ or $\frac{2}{6}>\frac{1}{6}$
+	$\frac{1}{3}+\frac{1}{6}=\frac{2}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$
−	$\frac{1}{3}-\frac{1}{6}=\frac{2}{6}-\frac{1}{6}=\frac{1}{6}$
·	$\frac{1}{3}\cdot\frac{1}{6}=\frac{1}{18}$

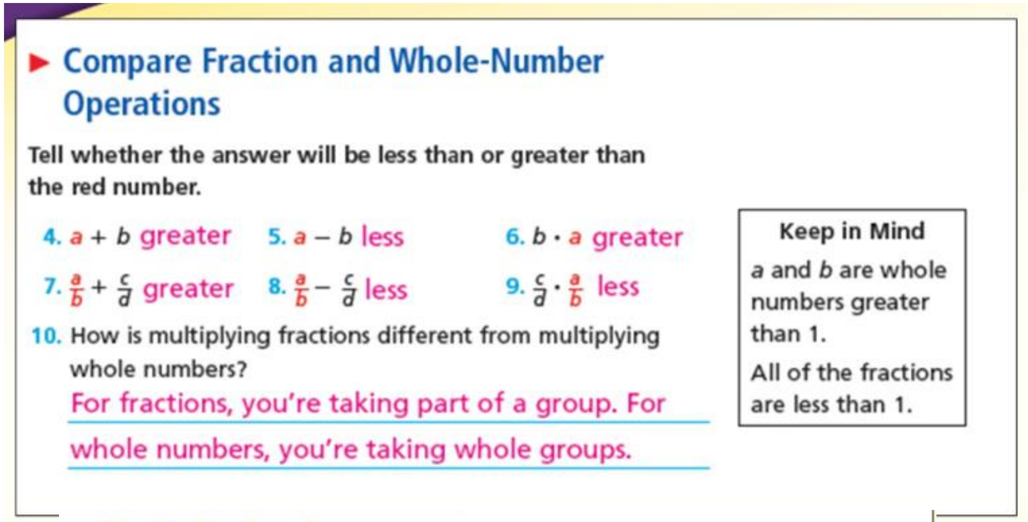
Complete the fraction box.

1.

	$\frac{2}{5}$ and $\frac{7}{10}$
>	$\frac{7}{10}>\frac{2}{5}$ or $\frac{7}{10}>\frac{4}{10}$
+	$\frac{7}{10}+\frac{2}{5}=\frac{7}{10}+\frac{4}{10}=\frac{11}{10}=1\frac{1}{10}$
−	$\frac{7}{10}-\frac{2}{5}=\frac{7}{10}-\frac{4}{10}=\frac{3}{10}$
·	$\frac{7}{10}\cdot\frac{2}{5}=\frac{14}{50}=\frac{7}{25}$

2.

	$\frac{3}{5}$ and $\frac{4}{7}$
>	$\frac{3}{5}>\frac{4}{7}$ or $\frac{21}{35}>\frac{20}{35}$
+	$\frac{3}{5}+\frac{4}{7}=\frac{21}{35}+\frac{20}{35}=\frac{41}{35}=1\frac{6}{35}$
−	$\frac{3}{5}-\frac{4}{7}=\frac{21}{35}-\frac{20}{35}=\frac{1}{35}$
·	$\frac{3}{5}\cdot\frac{4}{7}=\frac{12}{35}$

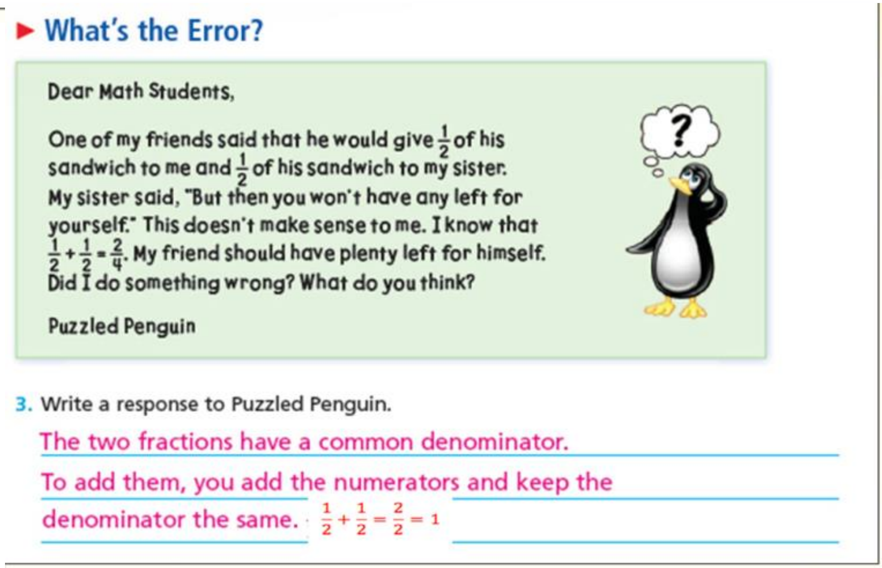


Compare Fraction and Whole-Number Operations

Tell whether the answer will be less than or greater than the red number.

4. $a + b$ greater
5. $a - b$ less
6. $b \cdot a$ greater
7. $\frac{a}{b}+\frac{c}{d}$ greater
8. $\frac{a}{b}-\frac{c}{d}$ less
9. $\frac{c}{d}\cdot\frac{a}{b}$ less
10. How is multiplying fractions different from multiplying whole numbers?
For fractions, you're taking part of a group. For whole numbers, you're taking whole groups.

Keep in Mind
a and b are whole numbers greater than 1.
All of the fractions are less than 1.



What's the Error?

Dear Math Students,

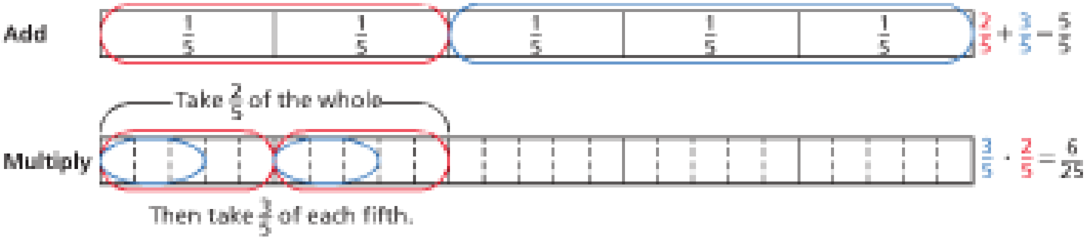
One of my friends said that he would give $\frac{1}{2}$ of his sandwich to me and $\frac{1}{2}$ of his sandwich to my sister. My sister said, "But then you won't have any left for yourself." This doesn't make sense to me. I know that $\frac{1}{2}+\frac{1}{2}=\frac{2}{4}$. My friend should have plenty left for himself. Did I do something wrong? What do you think?

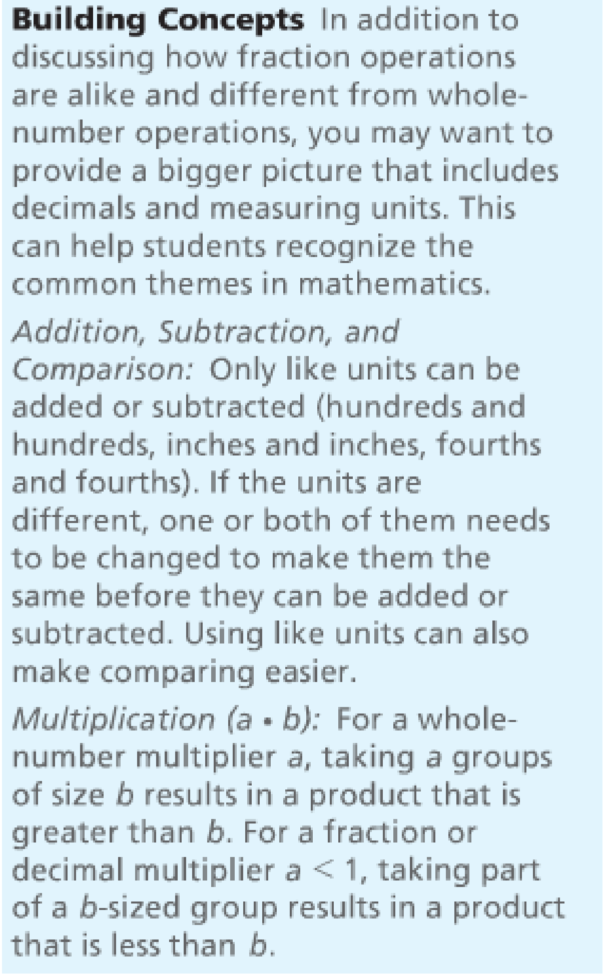
Puzzled Penguin

3. Write a response to Puzzled Penguin.
The two fractions have a common denominator. To add them, you add the numerators and keep the denominator the same. $\frac{1}{2}+\frac{1}{2}=\frac{2}{2}=1$

FRACTIONAL OPERATIONS

- Adding $\frac{2}{3}$ *to* $\frac{1}{3}$
  - 1 third plus 2 thirds
  - Greater than $\frac{1}{3}$, putting together
- Taking $\frac{2}{3}$ *of* $\frac{1}{3}$
  - 2 thirds of 1 third
  - less than $\frac{1}{3}$, taking apart
- Discuss Building Concepts





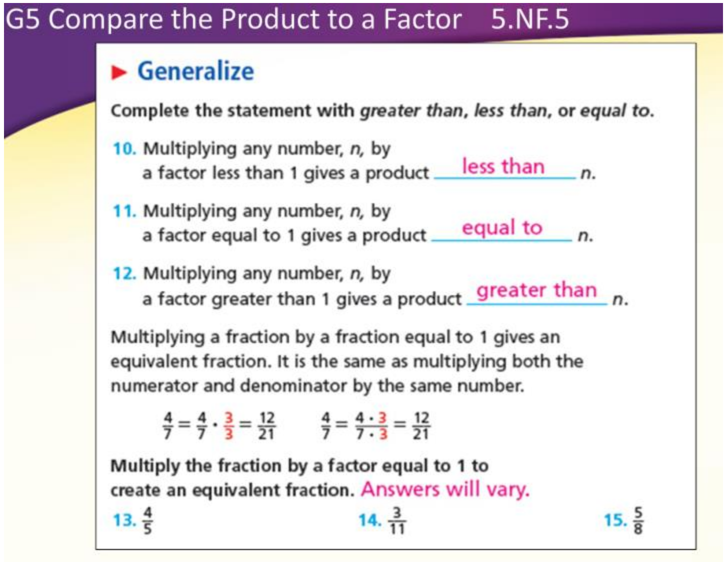
Building Concepts In addition to discussing how fraction operations are alike and different from whole-number operations, you may want to provide a bigger picture that includes decimals and measuring units. This can help students recognize the common themes in mathematics.

Addition, Subtraction, and Comparison: Only like units can be added or subtracted (hundreds and hundreds, inches and inches, fourths and fourths). If the units are different, one or both of them needs to be changed to make them the same before they can be added or subtracted. Using like units can also make comparing easier.

Multiplication (a • b): For a whole-number multiplier *a*, taking *a* groups of size *b* results in a product that is greater than *b*. For a fraction or decimal multiplier *a* < 1, taking part of a *b*-sized group results in a product that is less than *b*.

VIDEO

- Multiply a whole number *w* by another whole number
  - Product will be greater than w because you are combining more than one copy of w
- Multiply a fraction $\frac{a}{b}$ that is less than 1 by another fraction less than 1
  - Product will be less than $\frac{a}{b}$ because you are taking a part of $\frac{a}{b}$



G5 Compare the Product to a Factor 5.NF.5

Generalize

Complete the statement with *greater than*, *less than*, or *equal to*.

10. Multiplying any number, *n*, by a factor less than 1 gives a product less than *n*.

11. Multiplying any number, *n*, by a factor equal to 1 gives a product equal to *n*.

12. Multiplying any number, *n*, by a factor greater than 1 gives a product greater than *n*.

Multiplying a fraction by a fraction equal to 1 gives an equivalent fraction. It is the same as multiplying both the numerator and denominator by the same number.

$$\frac{4}{7} = \frac{4}{7} \cdot \frac{3}{3} = \frac{12}{21} \qquad \frac{4}{7} = \frac{4 \cdot 3}{7 \cdot 3} = \frac{12}{21}$$

Multiply the fraction by a factor equal to 1 to create an equivalent fraction. Answers will vary.

13. $\frac{4}{5}$

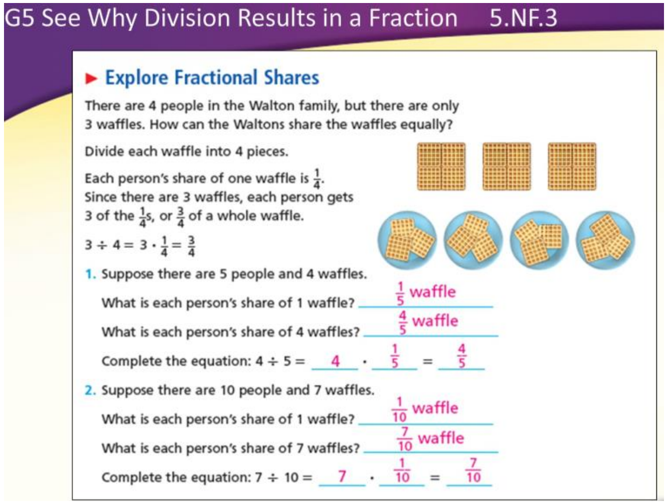
14. $\frac{3}{11}$

15. $\frac{5}{8}$

VIDEO

- Relate multiplication and division
  - $4 \cdot \frac{3}{4} = 3$ $3 \div 4 = \frac{3}{4}$
  - F F P P F F
  - Solving 3 ÷ 4 = ? Is equivalent to 4 ·___= 3

  - $12 \cdot \frac{1}{2} = 6$ $6 \div \frac{1}{2} = 12$
  - F F P P F F
  - Discuss the inverse relationship between the equations
  - First equation tells us that if we combine 12 groups of $\frac{1}{2}$ we get 6
  - The second tells us that if we divide 6 into groups of $\frac{1}{2}$ we get 12 groups



G5 See Why Division Results in a Fraction 5.NF.3

Explore Fractional Shares

There are 4 people in the Walton family, but there are only 3 waffles. How can the Waltons share the waffles equally?

Divide each waffle into 4 pieces.

Each person's share of one waffle is $\frac{1}{4}$. Since there are 3 waffles, each person gets 3 of the $\frac{1}{4}$s, or $\frac{3}{4}$ of a whole waffle.

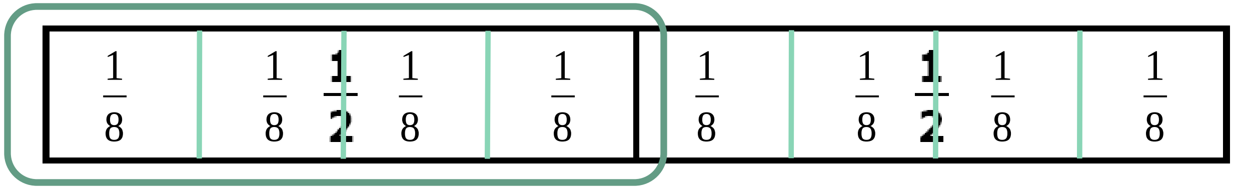
$3 \div 4 = 3 \cdot \frac{1}{4} = \frac{3}{4}$

1. Suppose there are 5 people and 4 waffles.
   What is each person's share of 1 waffle? $\frac{1}{5}$ waffle
   What is each person's share of 4 waffles? $\frac{4}{5}$ waffle
   Complete the equation: $4 \div 5 =$ 4 · $\frac{1}{5}$ = $\frac{4}{5}$
2. Suppose there are 10 people and 7 waffles.
   What is each person's share of 1 waffle? $\frac{1}{10}$ waffle
   What is each person's share of 7 waffles? $\frac{7}{10}$ waffle
   Complete the equation: $7 \div 10 =$ 7 · $\frac{1}{10}$ = $\frac{7}{10}$

DIVIDING WITH UNIT FRACTIONS

- Please watch and reflect on the meaning of ½ ÷ 4.

  - Divide ½ into 4 equal groups…how much does each group equal?



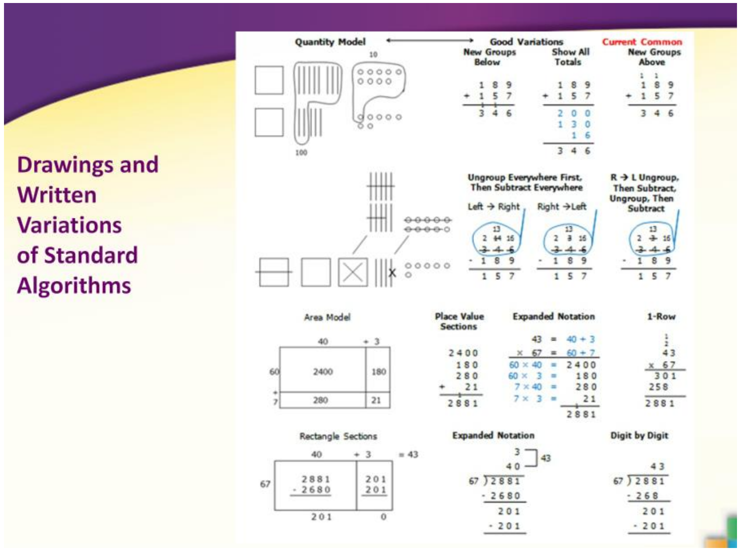
  - Each group equals 1/8.

IF TIME ALLOWS

- Multiplication and Division of whole numbers

MULTIPLICATION

- Array and area diagrams to represent multiplication
- Connect math drawings to numbers and symbols
- Algorithms are summaries of their reasoning about quantities



Drawings and Written Variations of Standard Algorithms

G4 Why So Many Grade 4 Multiplication Methods?

Math Expressions shows three methods to write each partial product:

a. Place Value Sections writes each partial product within rectangle sections and then adds these up outside; this is easier spatially for some students.

b. Expanded Notation writes a multiplication and the partial products with helping steps; the helping steps can be dropped whenever students can do so.

c. Algebraic Notation is like polynomial multiplication (40+3)(70+8); many advanced students love to "be doing algebra."

These methods all use an array/area rectangle model, show the distributive property in different ways, and lead students to deep understandings as the methods are related. Students build fluency with the method of their choice.

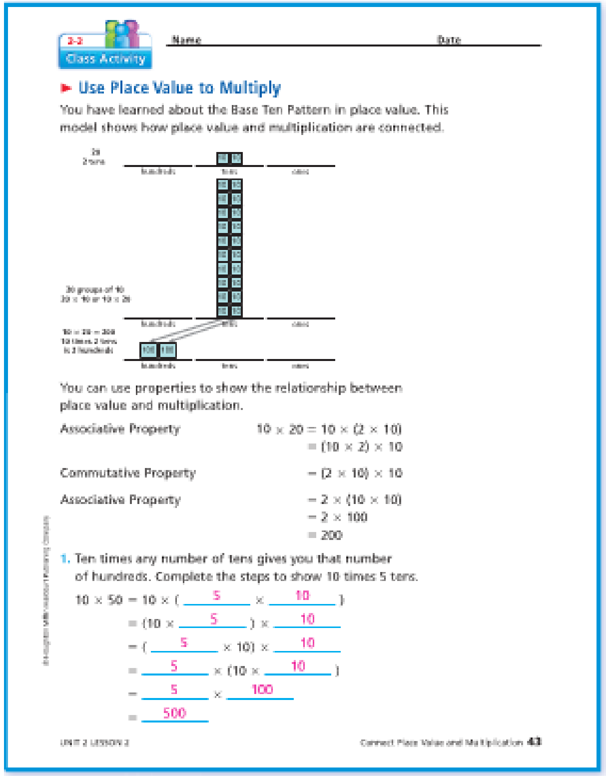
Students discuss more compact methods with two products each in a row. These methods are developed more fully in Grade 5 because they can be useful for G5 division by 2-digit numbers. These methods are difficult for many Grade 4 students for larger numbers and do not need to be mastered.

PLACE VALUE AND MULTIPLICATION

- Structure
  - Make connections between place value and multiplication

- Repeated reasoning
  - Generalize that 10 times any ones number gives you that number of tens and 10 times any hundreds number gives you that number of thousands

- This is the underlying concept upon which our place value system is built



2-2 Class Activity

Name ______________________ Date __________

Use Place Value to Multiply

You have learned about the Base Ten Pattern in place value. This model shows how place value and multiplication are connected.

You can use properties to show the relationship between place value and multiplication.

Associative Property $10 \times 20 = 10 \times (2 \times 10)$

$= (10 \times 2) \times 10$

Commutative Property $= (2 \times 10) \times 10$

Associative Property $= 2 \times (10 \times 10)$

$= 2 \times 100$

$= 200$

1. Ten times any number of tens gives you that number of hundreds. Complete the steps to show 10 times 5 tens.

$10 \times 50 = 10 \times (\underline{\ 5\ } \times \underline{\ 10\ })$

$= (10 \times \underline{\ 5\ }) \times \underline{\ 10\ }$

$= (\underline{\ 5\ } \times 10) \times \underline{\ 10\ }$

$= \underline{\ 5\ } \times (10 \times \underline{\ 10\ })$

$= \underline{\ 5\ } \times \underline{\ 100\ }$

$= \underline{\ 500\ }$

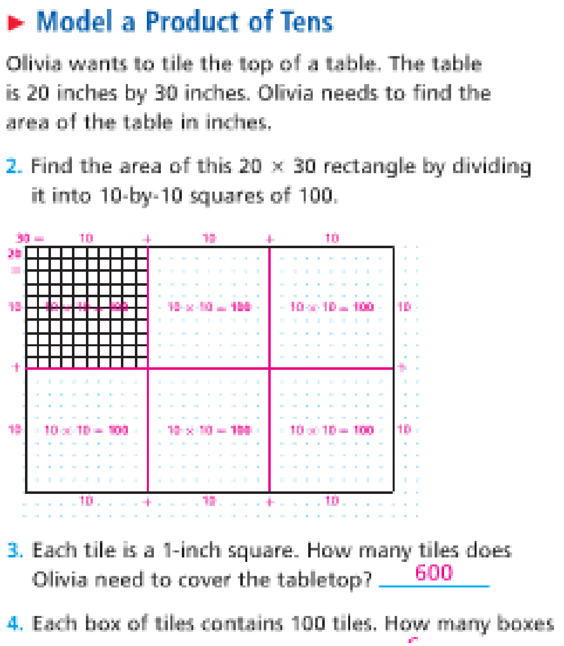
UNIT 2 LESSON 2

Connect Place Value and Multiplication 43

ZERO PATTERNS

Make Sense, Model, and Structure

- Draw a 20 x 30 rectangle on your MathBoard
- Divide that rectangle into 10-by-10 squares
  - What do the smaller boxes represent?
  - Notice the tiles are set in 20 equal groups of 30
  - What value does each of these squares represent?
  - How many groups of 100 are there?
  - What is the total area?



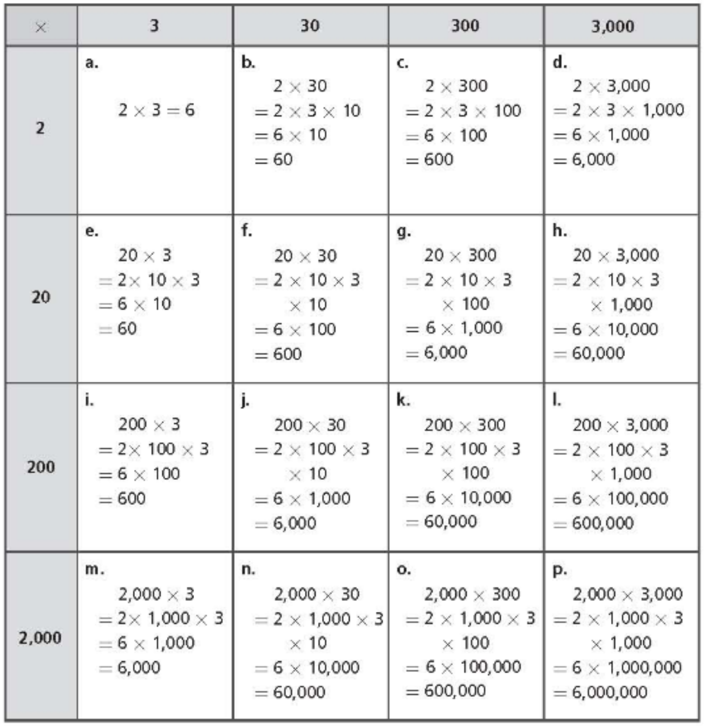
▶ Model a Product of Tens

Olivia wants to tile the top of a table. The table is 20 inches by 30 inches. Olivia needs to find the area of the table in inches.

2. Find the area of this 20 × 30 rectangle by dividing it into 10-by-10 squares of 100.

3. Each tile is a 1-inch square. How many tiles does Olivia need to cover the tabletop? 600

4. Each box of tiles contains 100 tiles. How many boxes



×	3	30	300	3,000
2	a. 2 × 3 = 6	b. 2 × 30 = 2 × 3 × 10 = 6 × 10 = 60	c. 2 × 300 = 2 × 3 × 100 = 6 × 100 = 600	d. 2 × 3,000 = 2 × 3 × 1,000 = 6 × 1,000 = 6,000
20	e. 20 × 3 = 2× 10 × 3 = 6 × 10 = 60	f. 20 × 30 = 2 × 10 × 3 × 10 = 6 × 100 = 600	g. 20 × 300 = 2 × 10 × 3 × 100 = 6 × 1,000 = 6,000	h. 20 × 3,000 = 2 × 10 × 3 × 1,000 = 6 × 10,000 = 60,000
200	i. 200 × 3 = 2× 100 × 3 = 6 × 100 = 600	j. 200 × 30 = 2 × 100 × 3 × 10 = 6 × 1,000 = 6,000	k. 200 × 300 = 2 × 100 × 3 × 100 = 6 × 10,000 = 60,000	l. 200 × 3,000 = 2 × 100 × 3 × 1,000 = 6 × 100,000 = 600,000
2,000	m. 2,000 × 3 = 2× 1,000 × 3 = 6 × 1,000 = 6,000	n. 2,000 × 30 = 2 × 1,000 × 3 × 10 = 6 × 10,000 = 60,000	o. 2,000 × 300 = 2 × 1,000 × 3 × 100 = 6 × 100,000 = 600,000	p. 2,000 × 3,000 = 2 × 1,000 × 3 × 1,000 = 6 × 1,000,000 = 6,000,000

FACTOR THE TENS

Reason abstractly and quantitatively & Precision

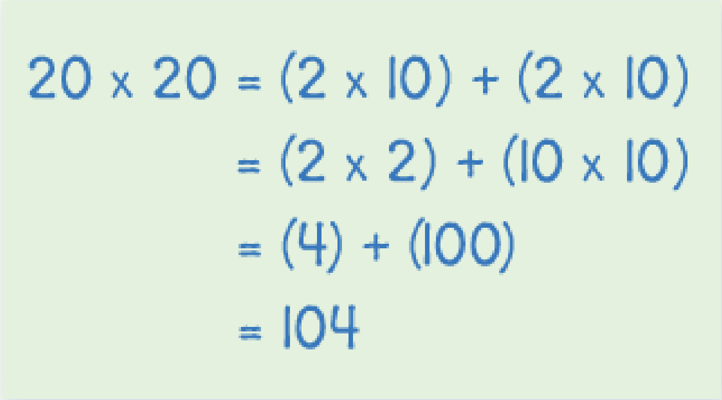
- Use place value language to explain where the numbers are coming from.
- Step one: factor
- Step two: commutative/associative property
- Step three: simplify
- Step four: product

2 tens x 3 tens

$$\begin{aligned} 20 \times 30 &= 2 \times 10 \times 3 \times 10 \\ &= (2 \times 3) \times (10 \times 10) \\ &= 6 \times 100 \\ &= 600 \end{aligned}$$

MATH TALK

- What’s the error?
  - Reason abstractly and quantitatively



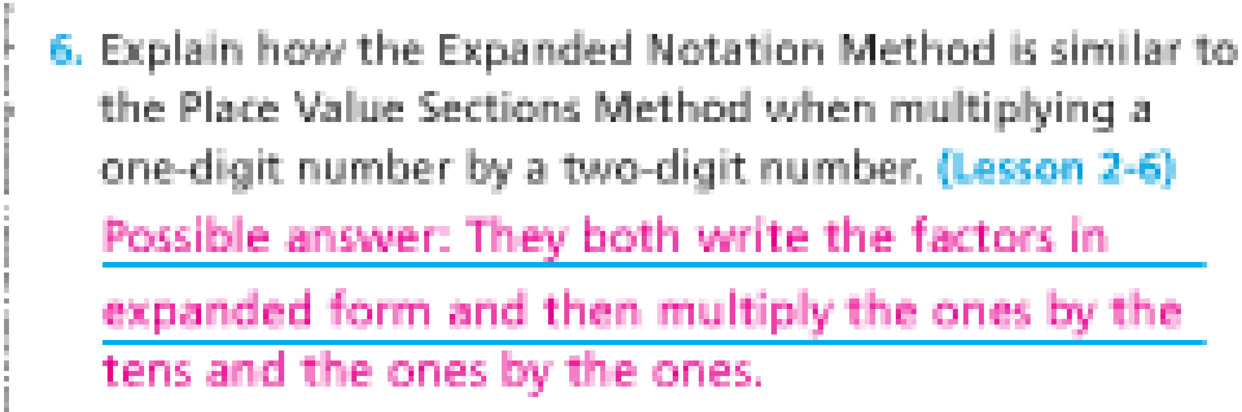
$$
\begin{aligned}
20 \times 20 &= (2 \times 10) + (2 \times 10) \\
&= (2 \times 2) + (10 \times 10) \\
&= (4) + (100) \\
&= 104
\end{aligned}
$$

- Added the factors instead of multiplying
- Drew a 20 x 20 rectangle and divided it into 10-by-10 squares of 100 the rectangle would show 4 groups of 100. That’s 400 square units not 104.

MULTIPLICATION METHODS

- Solve and discuss
  - Place Value Sections
  - Expanded Notation
  - Algebraic Notation
  - Shortcut

Make sure nouns and verbs match numerals and symbols

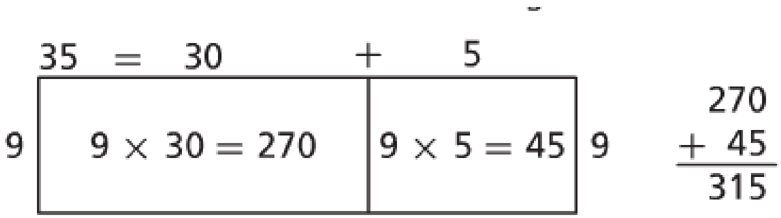


6. Explain how the Expanded Notation Method is similar to the Place Value Sections Method when multiplying a one-digit number by a two-digit number. (Lesson 2-6)

Possible answer: They both write the factors in expanded form and then multiply the ones by the tens and the ones by the ones.

PLACE VALUE SECTIONS METHOD

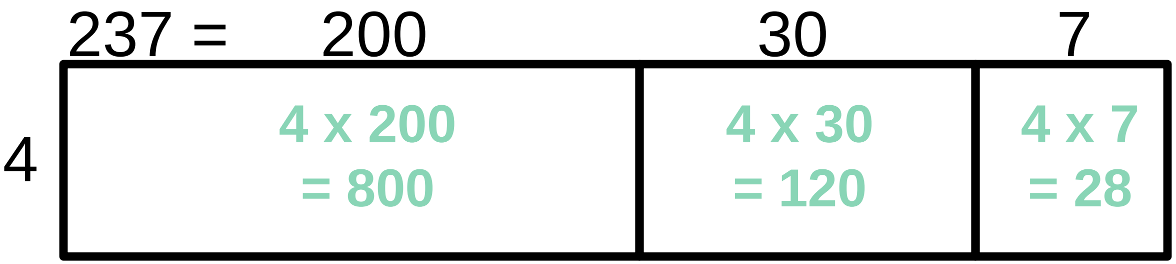
- Shows how to use the area model to multiply by recording each step inside the rectangle, then adding the area of each section outside the rectangle.



- Look for structure by describing what the sections represent
  - Left section shows the ones times the tens: 9 x 30
  - Right section shows the ones times the ones: 9 x 5

PLACE VALUE SECTIONS METHOD

- You may want to lead this exploration with all students working on a whiteboard.

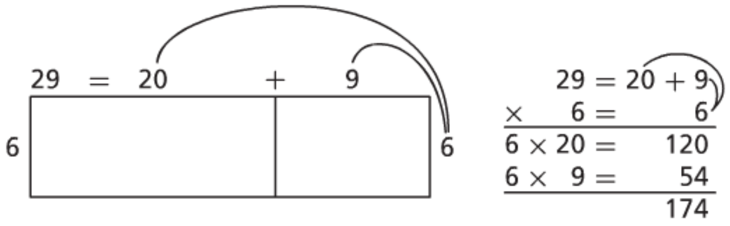


$$\begin{array}{r} 800 \\ 120 \\ +\ 28 \\ \hline 948 \end{array}$$

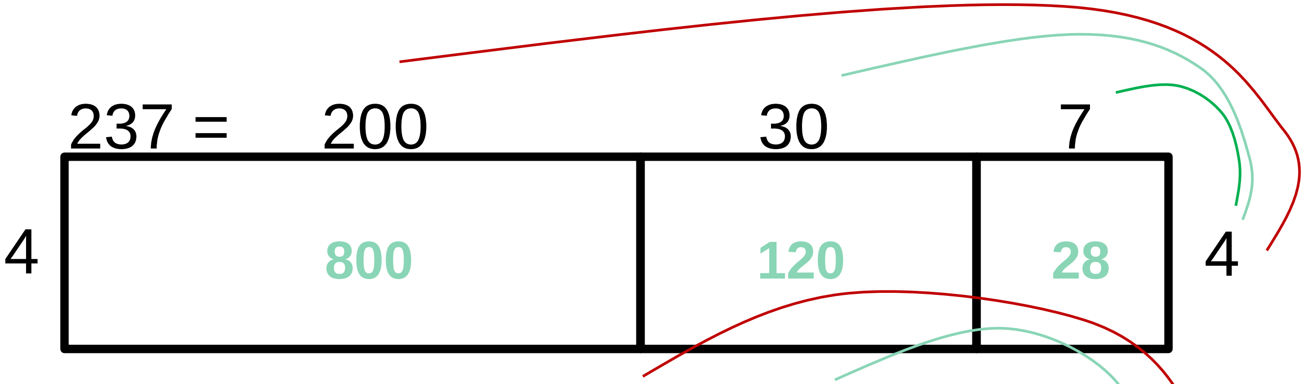
1. What are the two steps used to find the product of 4 x 237 using the Place Value Sections Method?

EXPANDED NOTATION METHOD

- Use the area method as a tool to explain expanded notation
  - How is the number 29 represented?
    - Remember that when a number shows the total value of each of its digits, it is written in expanded form
  - Relate the rectangular model to the numerical form by writing the expanded form of 29
  - Find the area of the tens section
    - Write the equation
  - Find the area of the ones section
    - Write the equation
  - Then add the two areas



EXPANDED NOTATION METHOD



237 =

	200	30	7	
4	800	120	28	4

237 = 200 + 30 + 7

x 4 = 4

---

4 x 200 = 800

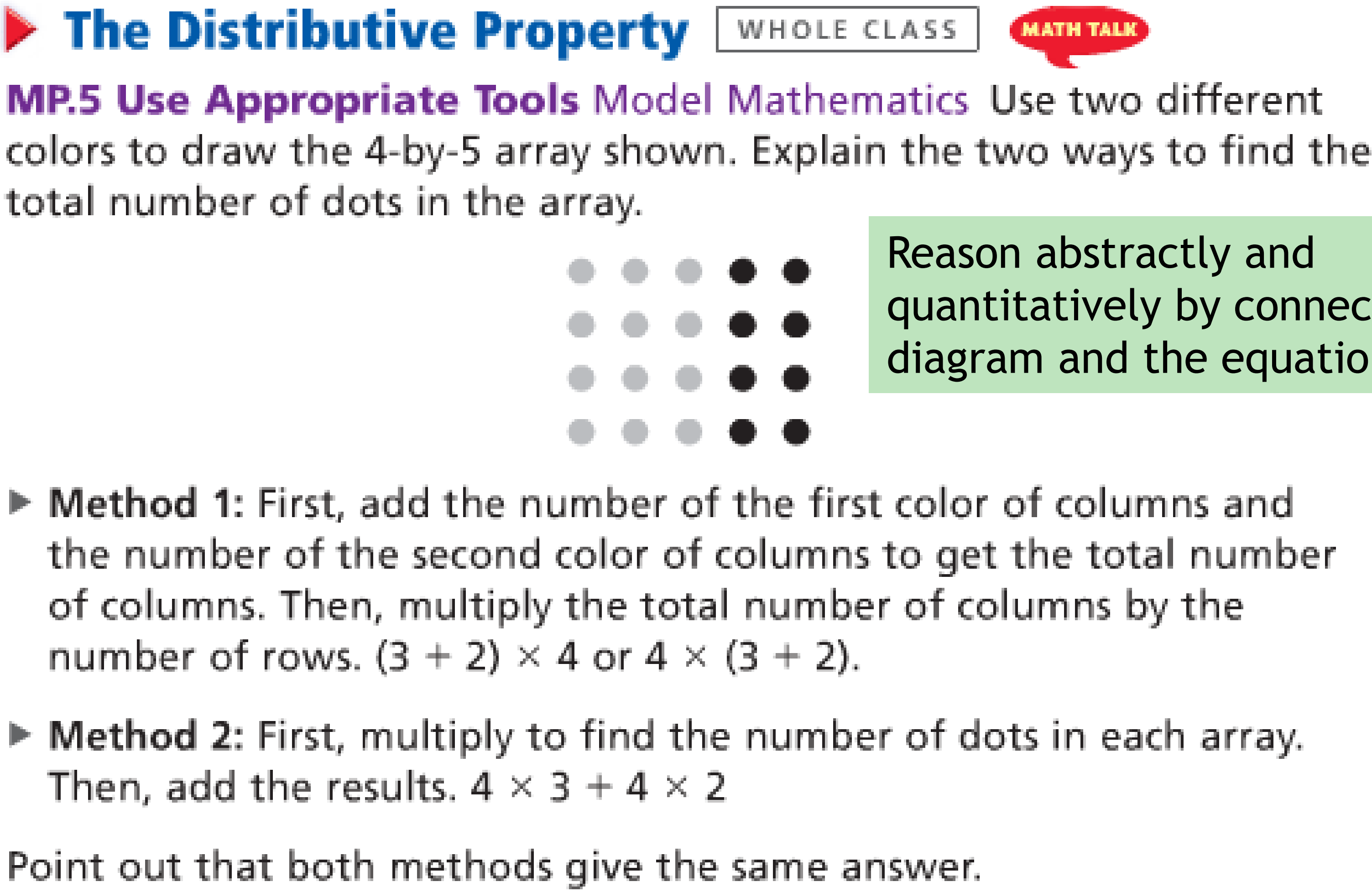
4 x 30 = 120

4 x 7 = 28

948

2. What is the last step in the Expanded Notation Method and the Place Value Sections Method?

DISTRIBUTIVE PROPERTY



The Distributive Property WHOLE CLASS MATH TALK

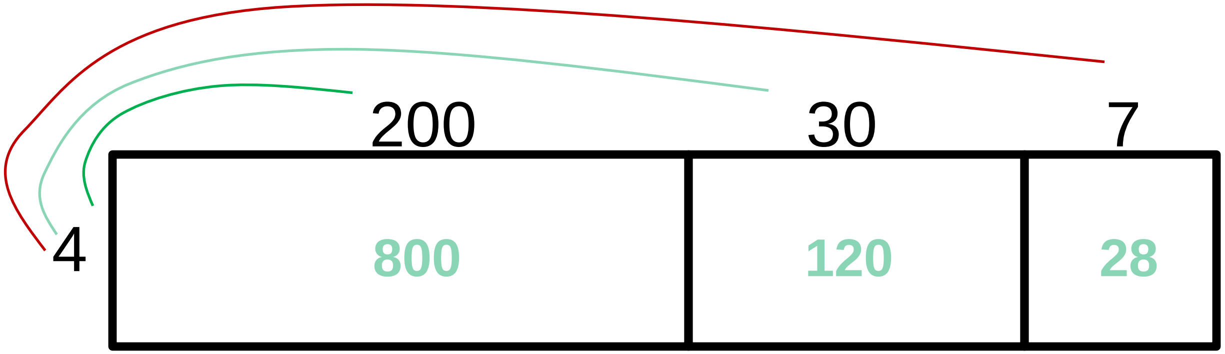
MP.5 Use Appropriate Tools Model Mathematics Use two different colors to draw the 4-by-5 array shown. Explain the two ways to find the total number of dots in the array.

- **Method 1:** First, add the number of the first color of columns and the number of the second color of columns to get the total number of columns. Then, multiply the total number of columns by the number of rows. (3 + 2) × 4 or 4 × (3 + 2).
- **Method 2:** First, multiply to find the number of dots in each array. Then, add the results. 4 × 3 + 4 × 2

Point out that both methods give the same answer.

Reason abstractly and quantitatively by connecting the diagram and the equation

ALGEBRAIC NOTATION METHOD



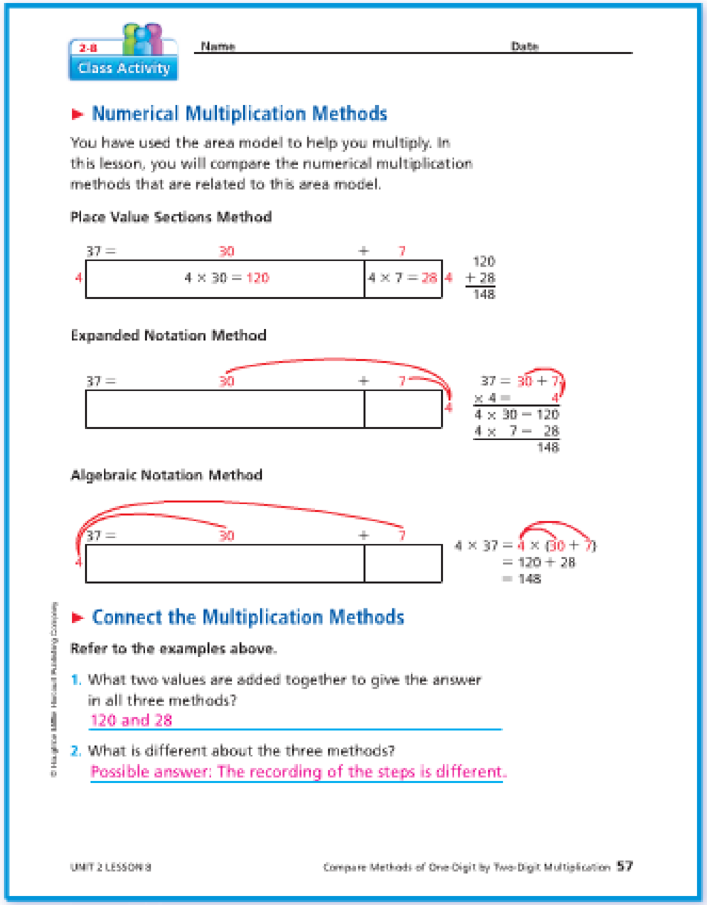
$$\begin{aligned} 4 \bullet 237 &= 4 \bullet (200 + 30 + 7) \\ &= 4 \bullet 200 + 4 \bullet 30 + 4 \bullet 7 \\ &= 800 + 120 + 28 \\ &= \mathbf{948} \end{aligned}$$

3. What is the first step in all three methods?

MATH TALK

Compare methods

- Which method do you prefer and why?
- Draw an area model and explain how it helps to solve the problem.
- Explain why the method you chose is different from the other two methods.



2-8 Class Activity

Name Date

Numerical Multiplication Methods

You have used the area model to help you multiply. In this lesson, you will compare the numerical multiplication methods that are related to this area model.

Place Value Sections Method

37 = 30 + 7

4 | 4 × 30 = 120 | 4 × 7 = 28 | 4

$$\begin{array}{r} 120 \\ +\ 28 \\ \hline 148 \end{array}$$

Expanded Notation Method

37 = 30 + 7

4

$$\begin{array}{r} 37 = 30 + 7 \\ \times\ 4 = \quad\ \ 4 \\ \hline 4 \times 30 = 120 \\ 4 \times\ \ 7 = \ \ 28 \\ \hline 148 \end{array}$$

Algebraic Notation Method

37 = 30 + 7

4

$$\begin{aligned} 4 \times 37 &= 4 \times (30 + 7) \\ &= 120 + 28 \\ &= 148 \end{aligned}$$

Connect the Multiplication Methods

Refer to the examples above.

1. What two values are added together to give the answer in all three methods?
   120 and 28
2. What is different about the three methods?
   Possible answer: The recording of the steps is different.

SHORTCUT METHOD

Must attend to precision and use structure

$$\begin{array}{rrr} 1 & 2 & \\ 2 & 3 & 7 \\ \times & & 4 \\ \hline 9 & 4 & 8 \end{array}$$

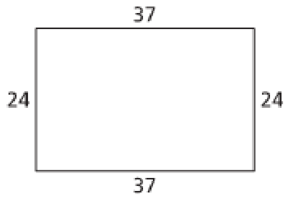
4 x 7 = 28, or 2 new tens and 8 ones

4 x 3 tens = 12 tens, plus 2 more tens is 14 tens, or a new hundred and 4 tens

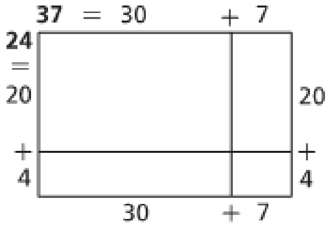
4 x 2 hundreds = 8 hundreds plus 1 more hundred is 9 hundreds.

DOUBLE DIGIT MULTIPLICATION

- How would you draw a model for 24 x 37?



- How can you show the tens and ones in 37?
  - Draw a vertical line and write 30 + 7
  - Draw a horizontal line and write 20 + 4



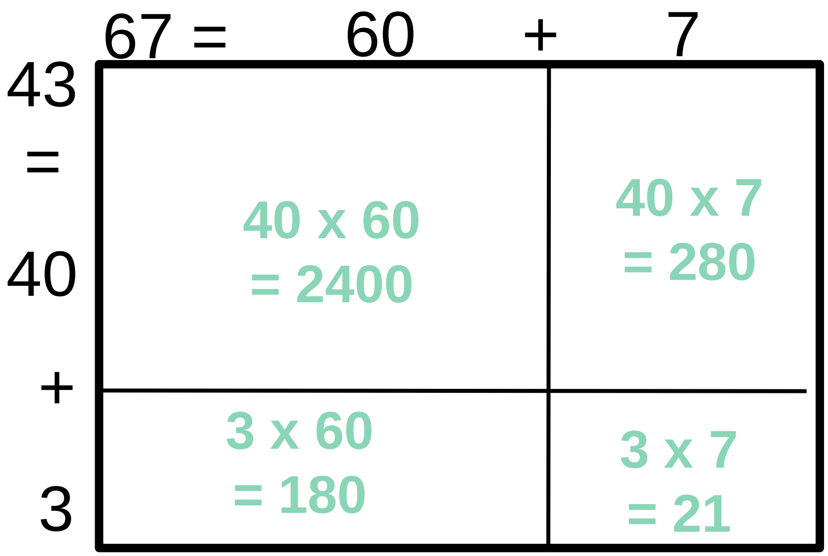
- Attend to precision and describe what each section represents
  - Record the equation for each step

- The top left section shows the *tens* times the *tens*: 20 × 30.
- The top right section shows the *tens* in 24 times the *ones* in 37: 20 × 7.
- The bottom left section shows the *ones* in 24 times the *tens* in 37: 4 × 30.
- The bottom right section shows the *ones* times the *ones*: 4 × 7.

CONNECT DIAGRAMS AND EQUATIONS

43 x 67

Area Model



67 = 60 + 7

43 = 40 + 3

	60	7
40	40 x 60 = 2400	40 x 7 = 280
3	3 x 60 = 180	3 x 7 = 21

Expanded

43 = 40 + 3

x 67 = 60 + 7

40 x 60 = 2400

40 x 7 = 280

3 x 60 = 180

\+ 3 x 7 = 21

2881

Results

43

x 67

2400

280

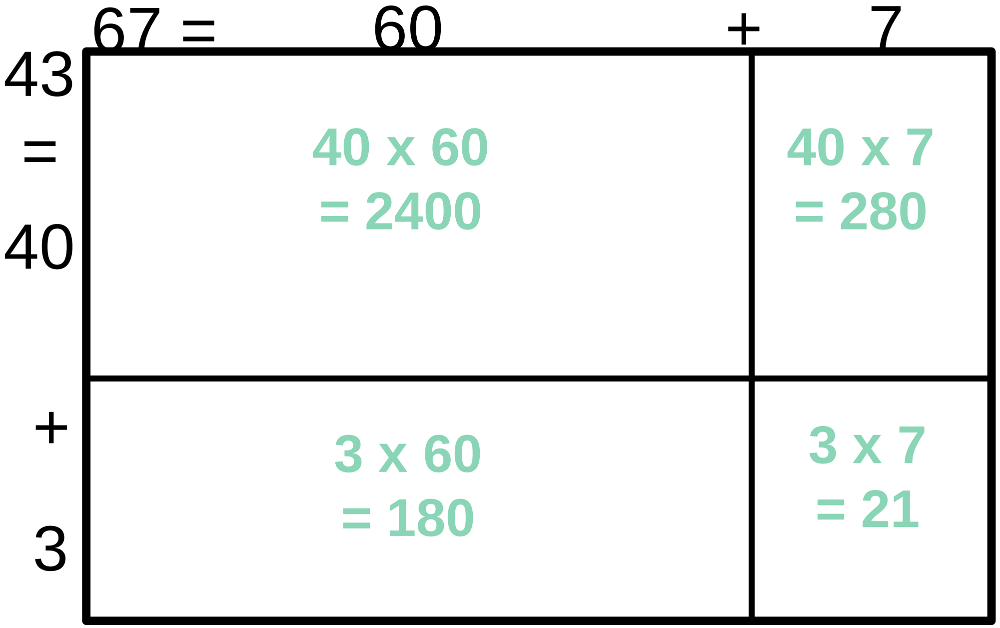
180

\+ 21

2881

Try, **28 x 54** at your table.

SHORTCUT METHOD



67 =	60	+ 7
43 = 40	40 x 60 = 2400	40 x 7 = 280
+ 3	3 x 60 = 180	3 x 7 = 21

$$\begin{array}{r} {\scriptstyle 2} \\ {\scriptstyle 2} \\ 6\ 7 \\ \times\ 4\ 3 \\ \hline 2\ 0\ 1 \\ +\ 2\ 6\ 8\ 0 \\ \hline 2,8\ 8\ 1 \end{array}$$

Step 1: 3 x 7 = 21, the 2 tens wait above the 6 tens

Step 2: 3 x 6 tens (and include two tens from 3 x 7): 18 tens + 2 tens = 20

Step 3: Write a zero as a place holder, so that in later steps the products of 40 will be in the right place

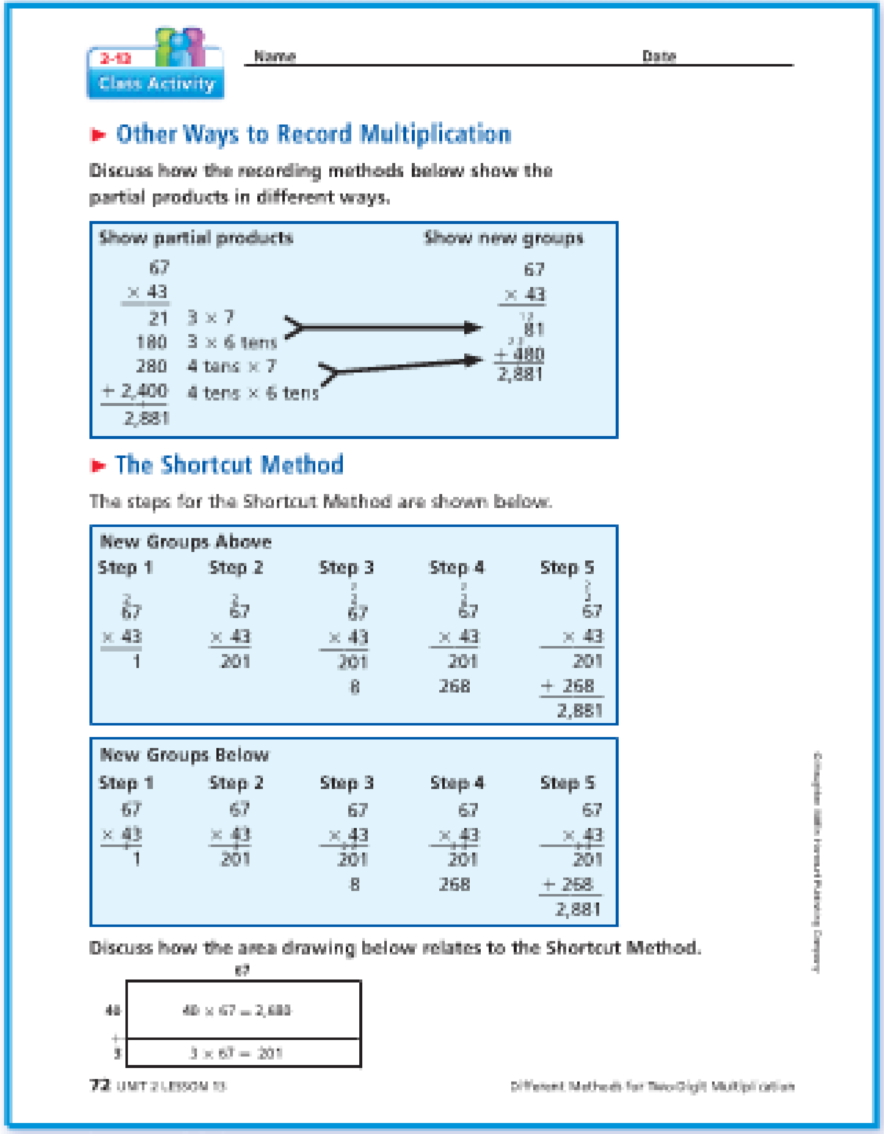
Step 4: 4 tens x 7 = 28 tens

Step 5: 4 tens x 6 tens (and included 2 hundreds from 4 tens x 7): 24 hundreds + 2 hundreds = 26 hundreds

Step 6: add the products of 3 x 67 and 40 x 67

OTHER WAYS TO RECORD MULTIPLICATION

- Partial products
- New groups above
- New groups below
- Shortcut
  - Use the area drawing to relate to the shortcut method



2-13 Class Activity

Name Date

Other Ways to Record Multiplication

Discuss how the recording methods below show the partial products in different ways.

Show partial products

67	
× 43	
21	3 × 7
180	3 × 6 tens
280	4 tens × 7
+ 2,400	4 tens × 6 tens
2,881	

Show new groups

67
× 43
81
+ 480
2,881

The Shortcut Method

The steps for the Shortcut Method are shown below.

New Groups Above

Step 1	Step 2	Step 3	Step 4	Step 5
67	67	67	67	67
× 43	× 43	× 43	× 43	× 43
1	201	201	201	201
		8	268	+ 268
				2,881

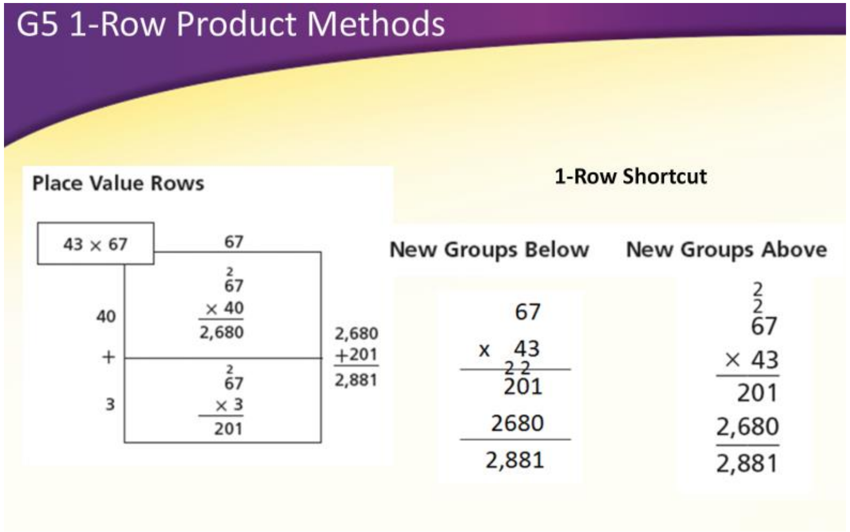
New Groups Below

Step 1	Step 2	Step 3	Step 4	Step 5
67	67	67	67	67
× 43	× 43	× 43	× 43	× 43
1	201	201	201	201
		8	268	+ 268
				2,881

Discuss how the area drawing below relates to the Shortcut Method.

MULTIPLICATION

- Methods
- Working toward fluency



G5 1-Row Product Methods

Place Value Rows

43 × 67

67

40 + 3

$\overset{2}{67}$ × 40 = 2,680

$\overset{2}{67}$ × 3 = 201

2,680
+201
2,881

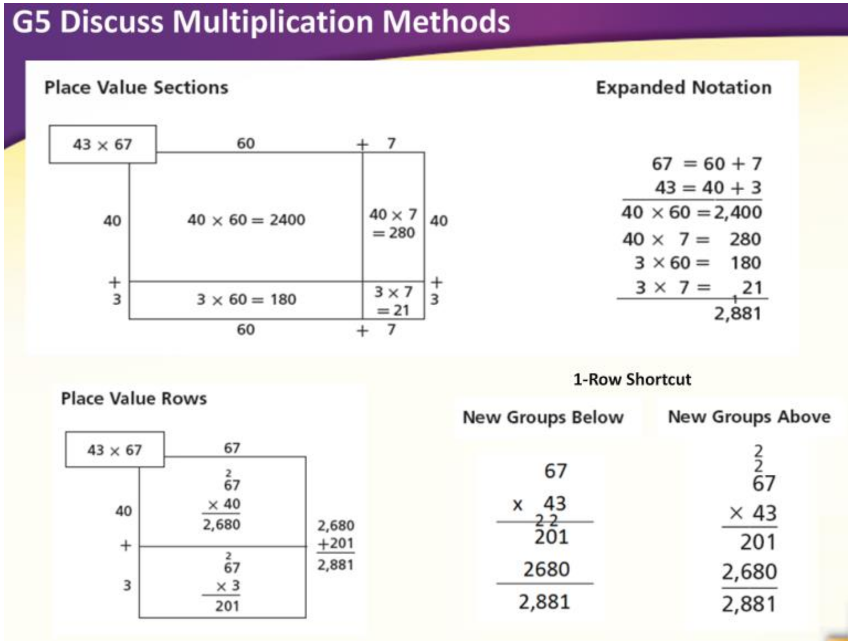
1-Row Shortcut

New Groups Below

67
x 43
2 2
201
2680
2,881

New Groups Above

2
2
67
× 43
201
2,680
2,881



G5 Discuss Multiplication Methods

Place Value Sections

43 × 67

60 + 7

40 + 3

40 × 60 = 2400

40 × 7 = 280

3 × 60 = 180

3 × 7 = 21

60 + 7

Expanded Notation

67 = 60 + 7
43 = 40 + 3
40 × 60 = 2,400
40 × 7 = 280
3 × 60 = 180
3 × 7 = 21
1
2,881

Place Value Rows

43 × 67

67

40 + 3

$\overset{2}{67}$ × 40 = 2,680

$\overset{2}{67}$ × 3 = 201

2,680
+201
2,881

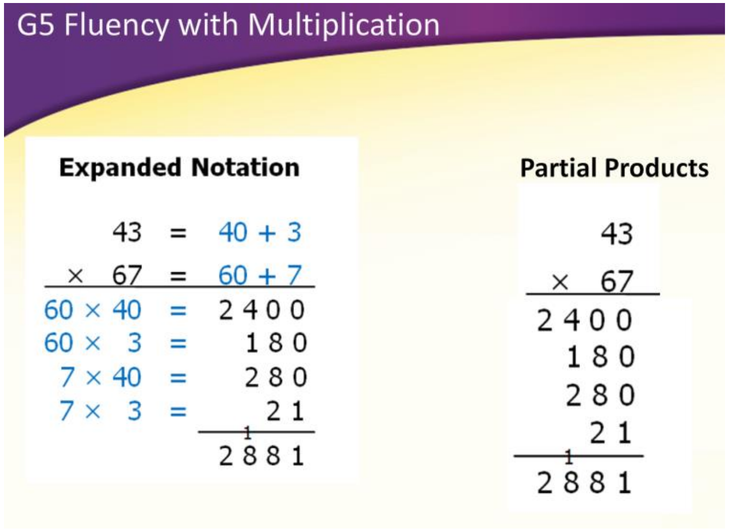
1-Row Shortcut

New Groups Below

67
x 43
2 2
201
2680
2,881

New Groups Above

2
2
67
× 43
201
2,680
2,881



G5 Fluency with Multiplication

Expanded Notation

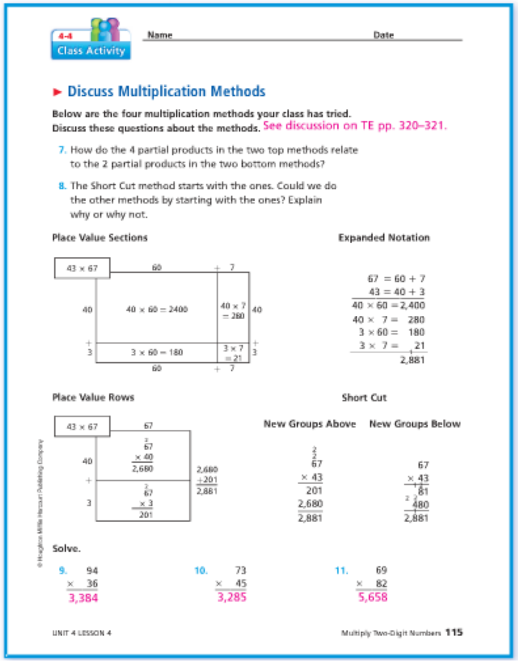
43 = 40 + 3
× 67 = 60 + 7
60 × 40 = 2 4 0 0
60 × 3 = 1 8 0
7 × 40 = 2 8 0
7 × 3 = 2 1
1
2 8 8 1

Partial Products

43
× 67
2 4 0 0
1 8 0
2 8 0
2 1
1
2 8 8 1

RELATE METHODS

- Please turn to the practice on your own.
- When finished, compare and discuss your work as a table group.



4-4 Class Activity

Name Date

Discuss Multiplication Methods

Below are the four multiplication methods your class has tried. Discuss these questions about the methods. See discussion on TE pp. 320–321.

7. How do the 4 partial products in the two top methods relate to the 2 partial products in the two bottom methods?
8. The Short Cut method starts with the ones. Could we do the other methods by starting with the ones? Explain why or why not.

Place Value Sections

Expanded Notation

67 = 60 + 7
43 = 40 + 3
40 × 60 = 2,400
40 × 7 = 280
3 × 60 = 180
3 × 7 = 21
2,881

Place Value Rows

Short Cut

New Groups Above **New Groups Below**

Solve.

9. 94 × 36 = 3,384
10. 73 × 45 = 3,285
11. 69 × 82 = 5,658

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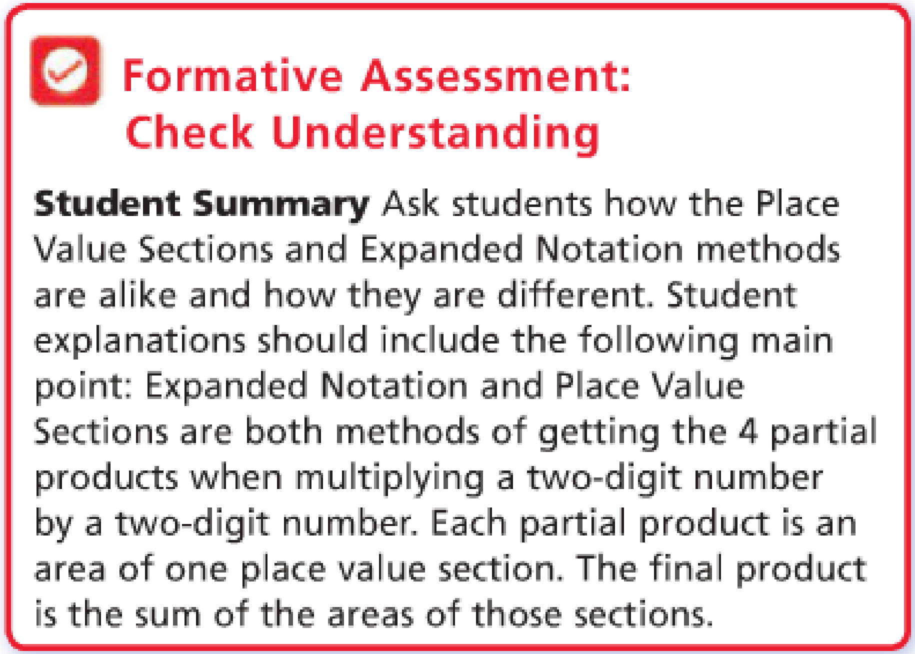
UNIT 4 LESSON 4 Multiply Two-Digit Numbers 115

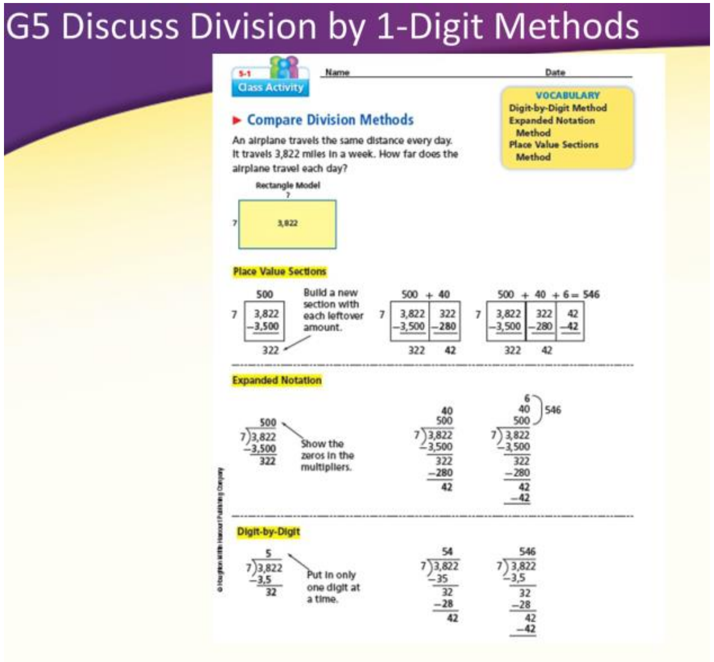


Formative Assessment: Check Understanding

Student Summary Ask students how the Place Value Sections and Expanded Notation methods are alike and how they are different. Student explanations should include the following main point: Expanded Notation and Place Value Sections are both methods of getting the 4 partial products when multiplying a two-digit number by a two-digit number. Each partial product is an area of one place value section. The final product is the sum of the areas of those sections.

DIVISION

- Compare methods
- Check for reasonableness
- 2 digit divisors



G5 Discuss Division by 1-Digit Methods

5-1 Class Activity

Name Date

VOCABULARY
Digit-by-Digit Method
Expanded Notation Method
Place Value Sections Method

Compare Division Methods

An airplane travels the same distance every day. It travels 3,822 miles in a week. How far does the airplane travel each day?

Place Value Sections

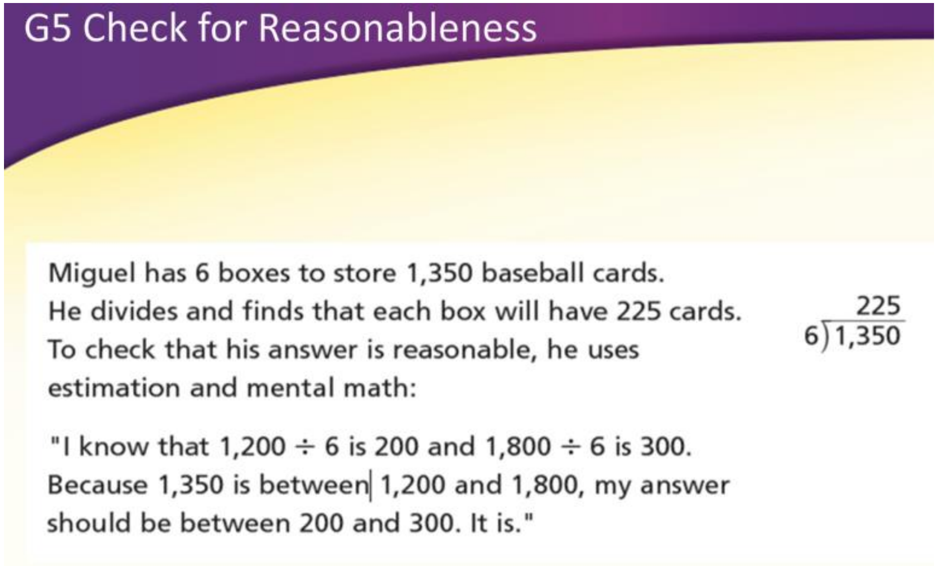
Build a new section with each leftover amount.

Expanded Notation

Show the zeros in the multipliers.

Digit-by-Digit

Put in only one digit at a time.

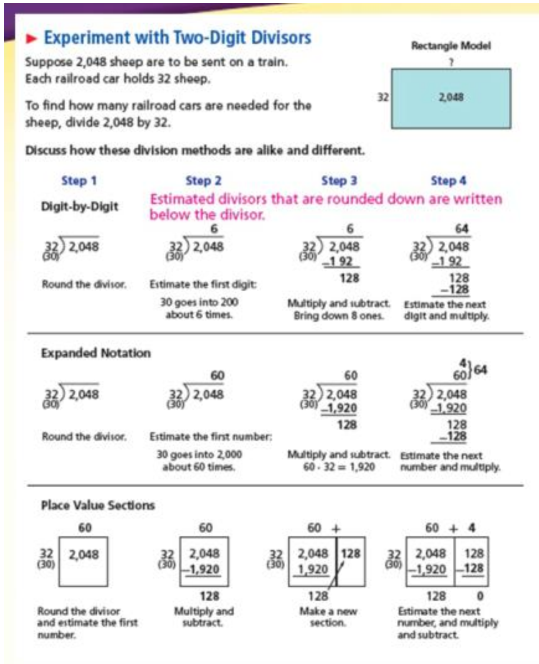


G5 Check for Reasonableness

Miguel has 6 boxes to store 1,350 baseball cards. He divides and finds that each box will have 225 cards. To check that his answer is reasonable, he uses estimation and mental math:

$$\begin{array}{r}225\\6\overline{)1,350}\end{array}$$

"I know that 1,200 ÷ 6 is 200 and 1,800 ÷ 6 is 300. Because 1,350 is between 1,200 and 1,800, my answer should be between 200 and 300. It is."



Experiment with Two-Digit Divisors

Suppose 2,048 sheep are to be sent on a train. Each railroad car holds 32 sheep.

To find how many railroad cars are needed for the sheep, divide 2,048 by 32.

Discuss how these division methods are alike and different.

Estimated divisors that are rounded down are written below the divisor.

Digit-by-Digit

Step 1: Round the divisor.
Step 2: Estimate the first digit: 30 goes into 200 about 6 times.
Step 3: Multiply and subtract. Bring down 8 ones.
Step 4: Estimate the next digit and multiply.

Expanded Notation

Round the divisor.
Estimate the first number: 30 goes into 2,000 about 60 times.
Multiply and subtract. 60 · 32 = 1,920
Estimate the next number and multiply.

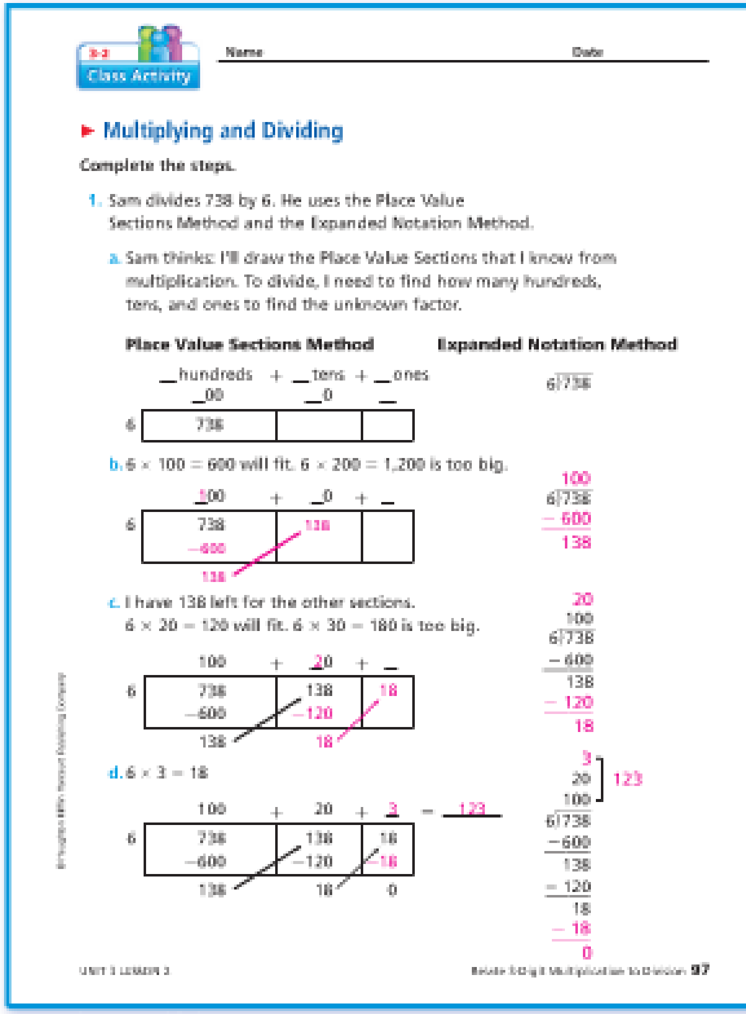
Place Value Sections

Round the divisor and estimate the first number.
Multiply and subtract.
Make a new section.
Estimate the next number, and multiply and subtract.

RELATE MULTIPLYING AND DIVIDING

Divide to find the number of groups in a division situation

- Place Value Sections Method
  - One section for each place value of the dividend
    - Multiplying with place value – relate dividend and divisor to factors and product
- Expanded Notation Method
  - Building quotient place value by stacking the sub-quotients



3-3 Class Activity

Name Date

Multiplying and Dividing

Complete the steps.

1. Sam divides 738 by 6. He uses the Place Value Sections Method and the Expanded Notation Method.

a. Sam thinks: I'll draw the Place Value Sections that I know from multiplication. To divide, I need to find how many hundreds, tens, and ones to find the unknown factor.

Place Value Sections Method **Expanded Notation Method**

b. 6 × 100 = 600 will fit. 6 × 200 = 1,200 is too big.

c. I have 138 left for the other sections.
6 × 20 = 120 will fit. 6 × 30 = 180 is too big.

d. 6 × 3 = 18

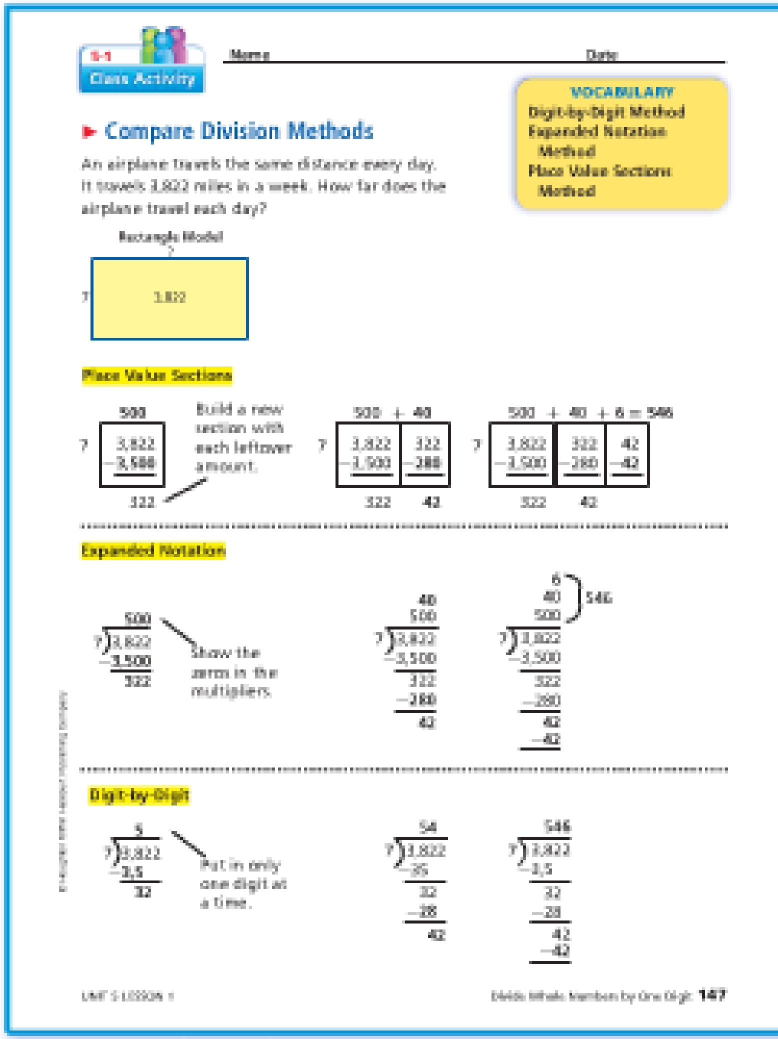
100 + 20 + 3 = 123

UNIT 3 LESSON 3

Relate 3-Digit Multiplication to Division 97

COMPARE METHODS

- Multiplication
  - Side lengths of the rectangle represent the factors
  - The area represents the unknown product
- Division
  - Area is given and one of the side lengths (one factor) is unknown
    - Area represents the dividend
    - One side length represents the divisor
    - The other represent the unknown quotient



5-1 Class Activity

Name Date

VOCABULARY
Digit-by-Digit Method
Expanded Notation Method
Place Value Sections Method

Compare Division Methods

An airplane travels the same distance every day. It travels 3,822 miles in a week. How far does the airplane travel each day?

Rectangle Model
?
7
3,822

Place Value Sections

500
7
3,822
−3,500
322

Build a new section with each leftover amount.

500 + 40
7
3,822 | 322
−3,500 | −280
322 | 42

500 + 40 + 6 = 546
7
3,822 | 322 | 42
−3,500 | −280 | −42
322 | 42

Expanded Notation

$$7\overline{)3{,}822}$$
500
−3,500
322

Show the zeros in the multipliers.

40
500
7)3,822
−3,500
322
−280
42

6
40
500
546
7)3,822
−3,500
322
−280
42
−42

Digit-by-Digit

5
7)3,822
−3,5
32

Put in only one digit at a time.

54
7)3,822
−35
32
−28
42

546
7)3,822
−3,5
32
−28
42
−42

UNIT 5 LESSON 1

Divide Whole Numbers by One Digit 147

DISTRIBUTIVE PROPERTY

- **Analyze Relationships: 325 ÷ 5**

Multiplication and division are inverse operations

325 ÷ 5 can be written as 5 x 65 = 325

5 x 65 = 5 x (60 + 5)

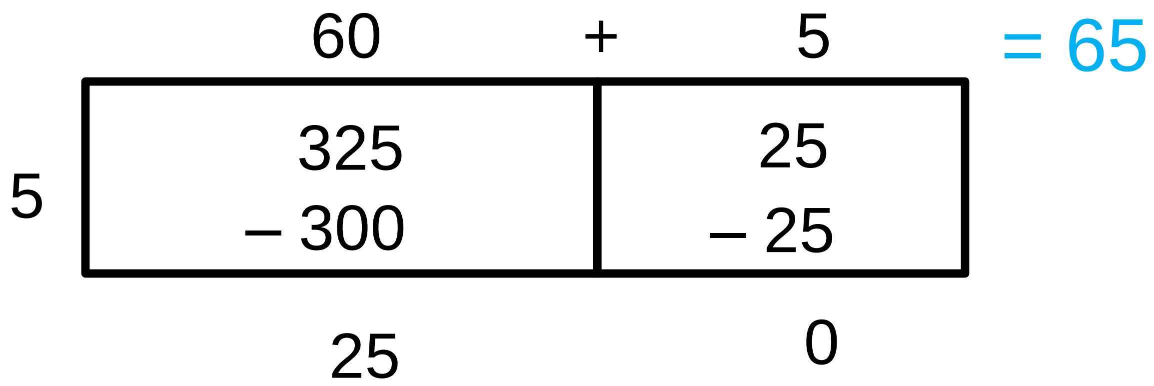
= (5 x 60) + (5 x 5)

= 300 + 25

= 325

Partial products 300 and 25 can be seen in the place value sections

Same as we subtract 300 in the first step and 25 in the last step



60 + 5 = 65

	60	5
5	325 – 300	25 – 25
	25	0

PLACE VALUE SECTIONS

3,248 ÷ 5

Question 1: “**5** times what **hundreds** number gives an answer closest to **3,248** without going over?”

Question 2: “**5** times what **tens** number gives an answer closest to **248** without going over?”

Question 3: “**5** times what number gives an answer closest to **48** without going over?”

	600 +	40 +	9	= 649 R3
5	3,248 – 3,000	248 – 200	48 – 45	
	248	48	3	

EXPANDED NOTATION

3,248 ÷ 5

Question 1: “**5** times what **hundreds** number gives an answer closest to **3,248** without going over?”

Question 2: “**5** times what **tens** number gives an answer closest to **248** without going over?”

Question 3: “**5** times what number gives an answer closest to **48** without going over?”

600 + 40 + 9 = 649 R3

5	3,248 – 3,000	248 – 200	48 – 45
	248	48	3

$$\left.\begin{array}{r}9\\40\\600\end{array}\right]\ 649\text{ R3}$$

$$\begin{array}{r} 5\overline{)\,3,248} \\ -\,3,000 \\ \hline 248 \\ -\,200 \\ \hline 48 \\ -\,45 \\ \hline 3 \end{array}$$

DIGIT BY DIGIT

3,248 ÷ 5

Question 1: “**5** times what number gives an answer closest to **3** without going over?”

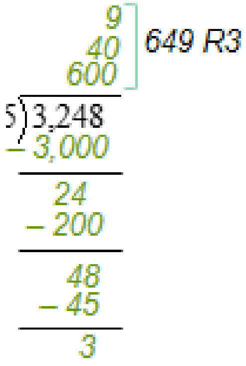
Question 2: “**5** times what number gives an answer closest to **32** without going over?”

Question 3: “**5** times what number gives an answer closest to **24** without going over?”

Question 4: “**5** times what number gives an answer closest to **48** without going over?”

600 + 40 + 9 = 649 R3

5	3,248 – 3,000	248 – 200	48 – 45
	248	48	3



9
40
600
649 R3
5)3,248
– 3,000
24
– 200
48
– 45
3

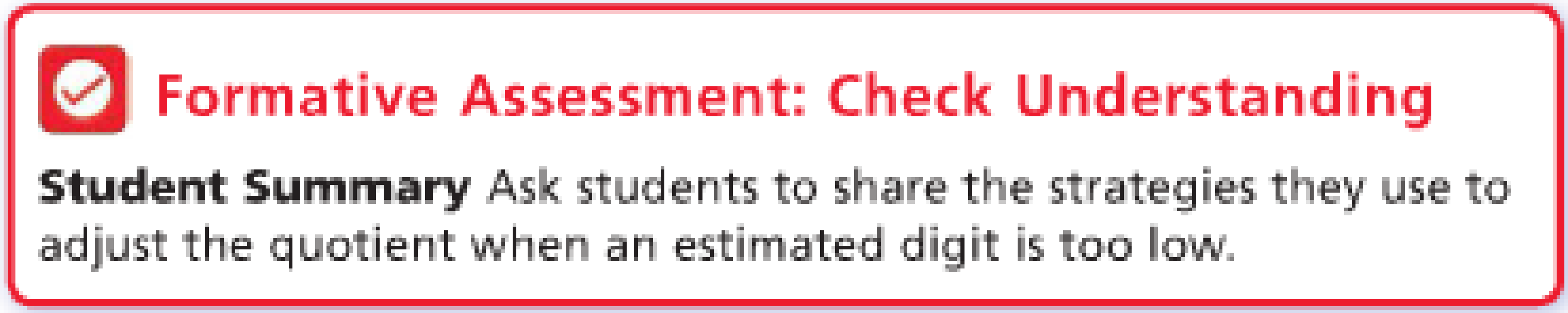
649 R3
5)3,248
– 30
24
– 20
48
– 45
3

COMPARING METHODS

- How are all three methods **similar**?
  - Build the unknown factor place-by-place.
  - Subtract each partial product from the part of the number being divided.
  - Write the unknown factor above the product with place values aligned.
- How is the digit-by-digit method **different**?
  - The digit-by-digit method only shows parts of the number being divided, instead of the complete partial products.
  - The digit-by-digit method, you write your answer as you go along, instead of having to add your partial answers at the end.

CHECK FOR UNDERSTANDING

- Discuss



Formative Assessment: Check Understanding

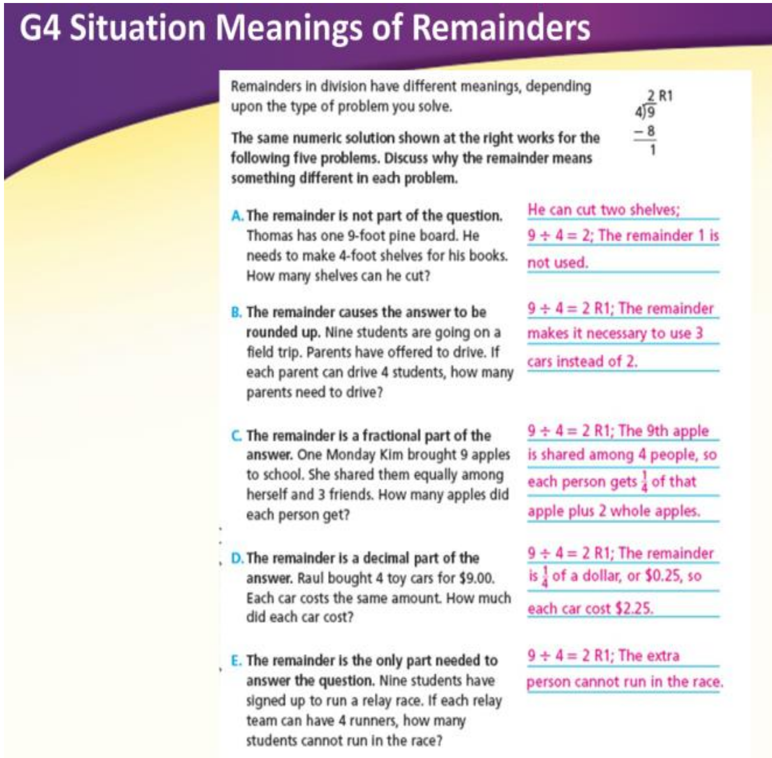
Student Summary Ask students to share the strategies they use to adjust the quotient when an estimated digit is too low.

Check or look up multiples on a multiplication table

- Learn difficult multiples
- Be less discouraged by division
- Find more accurate answers
- Recognize numbers that are far from an exact multiple

VIDEO: INTERPRET REMAINDERS

- Important to understand the meaning and usage of all types of remainders
  - A- Remainder that is not part of the question
  - B- Remainder that causes the answer to be rounded up
  - C- Fraction remainder
  - D- Decimal remainder
  - E- Remainder only



G4 Situation Meanings of Remainders

Remainders in division have different meanings, depending upon the type of problem you solve.

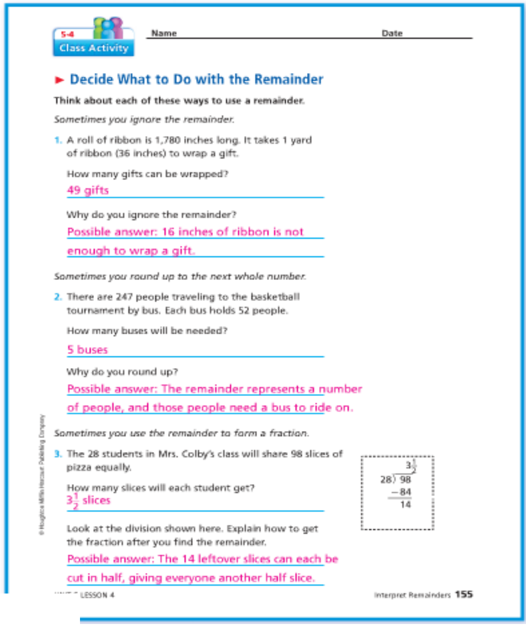
The same numeric solution shown at the right works for the following five problems. Discuss why the remainder means something different in each problem.

$$\begin{array}{r} 2\text{ R1} \\ 4\overline{)9} \\ -8 \\ \hline 1 \end{array}$$

A. **The remainder is not part of the question.** Thomas has one 9-foot pine board. He needs to make 4-foot shelves for his books. How many shelves can he cut?	He can cut two shelves; $9 \div 4 = 2$; The remainder 1 is not used.
B. **The remainder causes the answer to be rounded up.** Nine students are going on a field trip. Parents have offered to drive. If each parent can drive 4 students, how many parents need to drive?	$9 \div 4 = 2$ R1; The remainder makes it necessary to use 3 cars instead of 2.
C. **The remainder is a fractional part of the answer.** One Monday Kim brought 9 apples to school. She shared them equally among herself and 3 friends. How many apples did each person get?	$9 \div 4 = 2$ R1; The 9th apple is shared among 4 people, so each person gets $\frac{1}{4}$ of that apple plus 2 whole apples.
D. **The remainder is a decimal part of the answer.** Raul bought 4 toy cars for $9.00. Each car costs the same amount. How much did each car cost?	$9 \div 4 = 2$ R1; The remainder is $\frac{1}{4}$ of a dollar, or $0.25, so each car cost $2.25.
E. **The remainder is the only part needed to answer the question.** Nine students have signed up to run a relay race. If each relay team can have 4 runners, how many students cannot run in the race?	$9 \div 4 = 2$ R1; The extra person cannot run in the race.

REMAINDERS

- Discuss problems 1-5



5-4 Class Activity

Name Date

Decide What to Do with the Remainder

Think about each of these ways to use a remainder.

Sometimes you ignore the remainder.

1. A roll of ribbon is 1,780 inches long. It takes 1 yard of ribbon (36 inches) to wrap a gift.

   How many gifts can be wrapped?

   49 gifts

   Why do you ignore the remainder?

   Possible answer: 16 inches of ribbon is not enough to wrap a gift.

Sometimes you round up to the next whole number.

2. There are 247 people traveling to the basketball tournament by bus. Each bus holds 52 people.

   How many buses will be needed?

   5 buses

   Why do you round up?

   Possible answer: The remainder represents a number of people, and those people need a bus to ride on.

Sometimes you use the remainder to form a fraction.

3. The 28 students in Mrs. Colby's class will share 98 slices of pizza equally.

   How many slices will each student get?

   $3\frac{1}{2}$ slices

   $$\begin{array}{r} 3\frac{1}{2} \\ 28\overline{)\,98} \\ -84 \\ \hline 14 \end{array}$$

   Look at the division shown here. Explain how to get the fraction after you find the remainder.

   Possible answer: The 14 leftover slices can each be cut in half, giving everyone another half slice.

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LESSON 4

Interpret Remainders **155**

1. *Ignore the Remainder:* When we divide the ribbon into yards, we get 49 yards with 16 inches left over. Because it takes an entire yard of ribbon to wrap a gift, 16 inches is useless. We drop the remainder, and the answer is 49 gifts.

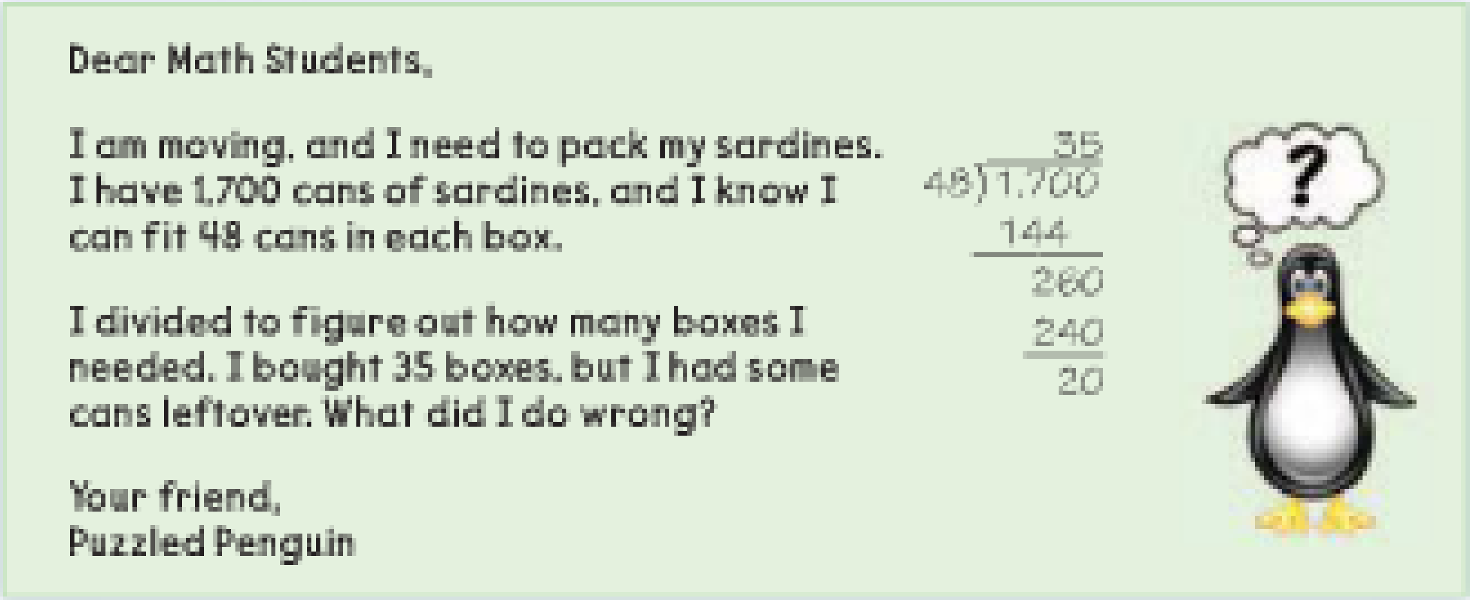
2. *Round Up:* When we divide the number of people into groups of 52, we get 4 groups with 39 people left over. These people cannot simply be left behind, so we add one more bus. The quotient is rounded up to 5, and the answer to the problem is 5 buses.

3. *Form a Fraction:* Unlike buses, slices of pizza can be split. Therefore, it makes sense to include a fraction in the answer. Each student gets 3 whole slices of pizza. The remainder is 14 slices. Because there are 28 students, the 14 slices can each be cut in half, giving everyone another half slice. Be sure the class sees that the remainder (14) becomes the numerator for the fraction, giving us $\frac{14}{28}$, which is $\frac{1}{2}$.

4. *Form a Decimal:* A part of a dollar (cents) is almost always expressed as a decimal number. Discuss the example about the car wash with the class. Then have students solve Problem 4. The answer can be written as the mixed number $25\frac{7}{35}$ m, or $25\frac{1}{5}$ m. However, because metric measurements are given in decimal form, we write this as 25.2 m.

5. *Use the Remainder Only:* Some situations require that we give the remainder only, not the number of groups or the size of the groups. Problem 5 asks how many bagels the workers will get, which is the remainder.

DISCUSS THE ERROR



Dear Math Students,

I am moving, and I need to pack my sardines. I have 1,700 cans of sardines, and I know I can fit 48 cans in each box.

I divided to figure out how many boxes I needed. I bought 35 boxes, but I had some cans leftover. What did I do wrong?

Your friend,
Puzzled Penguin

$$\begin{array}{r} 35 \\ 48\overline{)1{,}700} \\ \underline{144} \\ 260 \\ \underline{240} \\ 20 \end{array}$$

- You forgot the remainder. The answer to your division problem is 35 with a remainder of 20. This means that if you put 48 cans in a box, you will fill 35 boxes, but have 20 cans left over. You need to get an extra box to put the extra cans in, so you need 36 boxes in all.

Questions?
Comments…

ROBYN SEIFERT DECKER
ULTRAMATHPD@GMAIL.COM