

MATH ACADEMY LOWER ELEMENTARY

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AGENDA

- ◎ Place Value
- ◎ Make-A-Ten
- ◎ Teen Numbers
- ◎ Solve 1 and 2 step word problems


MATH PRACTICES

- ⦿ A teacher every day asks:
- ⦿ Did I do **math sense-making** about **math structure** using **math drawings** to support **math explaining**?
- ⦿ Can I do some part of this better tomorrow?

MULTIUNIT CONCEPTUAL CONNECTIONS


- **Multiunit Quantities**
 - Drawings
 - Sketches
- **English Number Words**
 - Named value
 - Expanded notation
- **Written Number Marks**
 - Concatenated single digit

Meanings for Drawings



Ten tens and one hundred ones One ten is ten ones.

MATH DRAWINGS (SKETCHES)



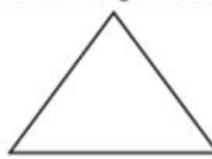
Three hundreds seven tens nine ones

QUANTITY WORDS

Sequence Multiunit Conception
Count the quantities by multiunit English named values within an accumulating sequence total: Count by hundreds, then tens, and then ones, shifting the count to each new multiunit: 100, 200, 300, (shift to tens) 310, 320, 330, 340, 350, 360, 370, (shift to ones) 371, 372, 373, 374, 375, 376, 377, 378, 379

Separate Multiunit Conception
Count (see) the multiunit quantities hundreds, tens, and ones

Multiunit Quantities



English Number Words
Named Value (Expanded Notation)

Written Number Marks
Concatenated Single Digit

SECRET CODE CARDS

300	70	9		300 70 9
3 0 0	7 0	9	=	3 7 9
				379

300 + 70 + 9 = 379

Three hundred seventy nine three seven nine

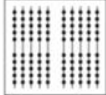
Learn the repeating cycles in the English counting sequence Learn to read left to right Position tells the value

IMPORTANCE OF CONCEPTUAL UNDERSTANDING


Common Errors

- Incorrectly translating zeros in the standard form of a number to the word form
- Forgetting to make a new group of hundreds when addition results in 10 tens
- When ungrouping hundreds to subtract, forgetting to reduce the number of hundreds by 1
- Not writing the correct addition equation to model and solve a real world problem

Meanings for Drawings




Ten tens and one hundred ones



One ten is ten ones.

MATH DRAWINGS (SKETCHES)



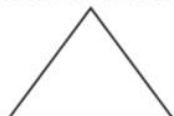
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QUANTITY WORDS

Sequence Multiunit Conception
Count the quantities by multiunit
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Count by hundreds, then tens, and then ones, shifting the count to each new multiunit: 100, 200, 300, (shift to tens) 310, 320, 330, 340, 350, 360, 370, (shift to ones) 371, 372, 373, 374, 375, 376, 377, 378, 379

Separate Multiunit Conception
Count (see) the multiunit quantities hundreds, tens, and ones


Multiunit Quantities



English Number Words
Named Value (Expanded Notation)

Written Number Marks
Concatenated Single Digit

SECRET CODE CARDS



300 + 70 + 9 = 379

Three hundred seventy nine

Learn the repeating cycles in the English counting sequence

three seven nine

Learn to read left to right
Position tells the value

PLACE VALUE LANGUAGE

Teen and 2-digit numbers are complex and difficult to learn

- 13 looks like 2 single digit numbers
- Verbal and visual support is needed to understand the ten value on the left and the ones digit on the right
- Visual supports for understanding the number words and written numbers is important
 - Use tens and ones words
 - Standard number words
 - Saying 13 as thirteen, 1 ten and 3 ones, 10 ones and 3 more ones
 - Reinforces embedded ten based thinking place value understanding
 - Words relate to the quantities kids make or count to show the ten ones and some further ones
 - Relate to the base-ten numerals in equations

TEN BUG



Structure and Precision

- Provide explanations of new concepts in kinder friendly ways
 - Students have 20 counters
 - Bug is trying to count to 20 and gets confused
 - Goal: students figure out ideas for counting to 20 without getting confused
 - Fingers, organized rows/columns
 - 10 groups
 - 5 groups
 - Dot-to-dot
 - Goal: build structure, seeing the number in the abstract

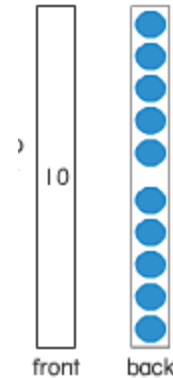
GROUP 10 TO MAKE TEEN NUMBERS

⦿ Ten counters

- Make 14 different ways
 - Neither the order or starting point matters
 - Always the same number of counters

⦿ Ten counter strip

- Shows 10 counters
- Able to make teen numbers quickly

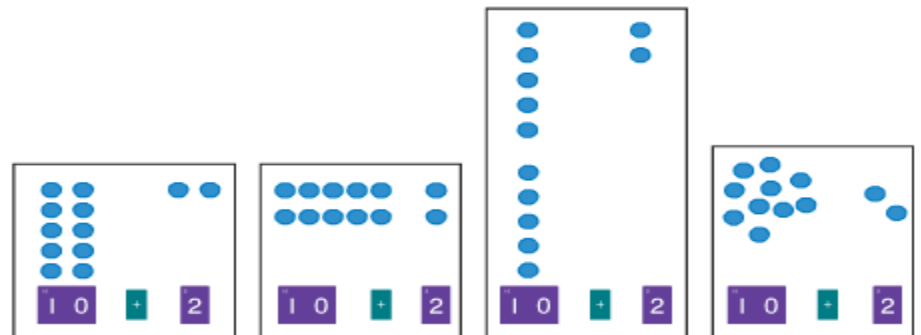


Focus is to see all of the teen numbers as ten ones and some further ones.

SEE TEN IN TEEN NUMBERS

Structure

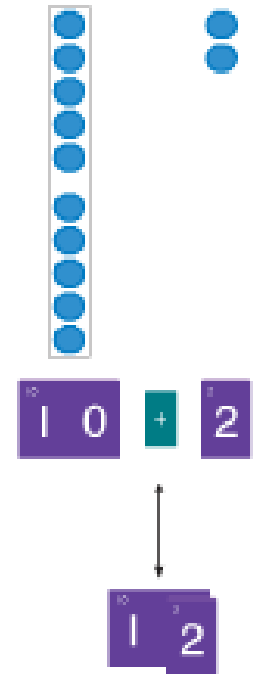
- Emphasis is on 10 group to be placed on the left with counters, strips, and number tiles
- Use the 10 number tile and a 2 number tile to show twelve
- Students build $10 + 2$ with tiles and counters



TEN COUNTERS WITH A 10-COUNTER STRIP

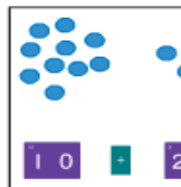
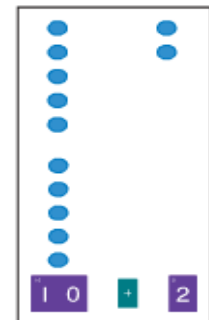
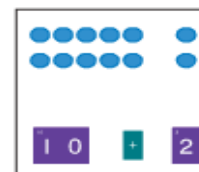
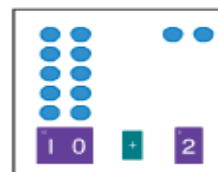
Teen Numbers as Tens and Ones

- Children will group 10 objects together to see the 10 “hiding” inside the teen number
 - Number tiles will be used to show this “hiding” ten by placing the one from the ten equation onto the ten
 - Expanded and Standard form of the number
 - 10 loose counters will be traded for various ten models, such as the 10-counter strip
 - Go back and forth between 12 and $10 + 2$ make twelve with tiles, finger freezes, 10-counter strips, and counters



SEE TENS IN TEENS

- Make ten with 5-square tiles
 - Similar to making a 10 with the counters and 10-counter strip
 - Goal: with dot side up students make a 10 quickly with two 5-strips
 - Students connect this to it being faster than 10 loose squares
 - Drawing numbers reinforces the value
 - Different arrangements of 5-groups is good for flexibility
 - BUT arranged groups are easier to count
 - Drawing vertical groups is harder
 - Must put a space between each 5



DEVELOPING CONCEPTS OF TEEN NUMBERS

Reasoning

Subitizing in 2 ways:

- Teen numbers are composed of ten ones and some further ones
 - Ten ones can be group together - which makes counting easier
 - This gets students ready to think of this group as a ready made ten
 - No need to count each unit individually
 - GRADE ONE FURTHER DEVELOPES THIS CONCEPT OF TEN AS A SINGLE UNIT - THIS IS THE FOUNDATION OF PLACE VALUE
- Teen number is the partner concept of ten ones and further ones
 - Teen numbers may be composed by adding two partner numbers

WATCH FOR!

- ⦿ Eleven and twelve
 - Do not follow the pattern of other numbers
- ⦿ Thirteen, fifteen
 - The ten or teen is the second part of the number that is said

TEN STRUCTURED TEENS

○ Elicit prior knowledge

- Introducing the base 10 system
- Teen numbers are 10s and some more 1s

○ Visualize

- Draw the 10 ones in a ten frame to connect that teen numbers have a ten - and so you can make a group of ten

○ Equation

- Structure of an equation is another way to explore representing teen numbers

► Model a Ten and Extra Ones WHOLE CLASS

Introduce tens-and-ones language. As you write 14 on the board, point out the tens place and the ones place.

| 4
tens ones

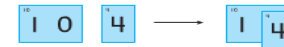
- The number 14 has 1 ten and 4 extra ones. Do you see the 1 in the tens place? Do you see the 4 in the ones place?

MP.5 Use Appropriate Tools *Secret Code Cards* Line up cards 1 through 10 on the ledge of the board, starting with the 10-card as shown below. Although the Demonstration Secret Code Cards have been used in the Quick Practice Routines in Unit 3, this lesson provides an opportunity for the class to explore the cards in more depth.

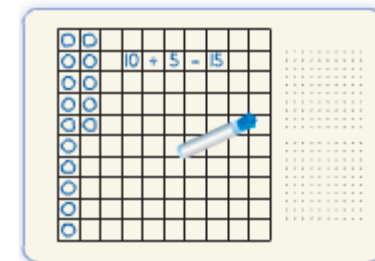
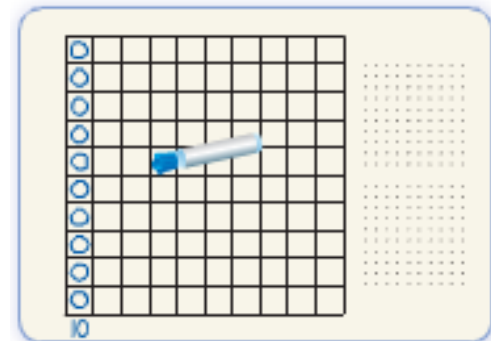


Tell children that teen numbers are the numbers 11 to 19 and that all teen numbers have 1 ten and some extra ones. Demonstrate how the Secret Code Cards can be used to make teen numbers by stacking two cards to show 14.

- I can make the number 14 with these Secret Code Cards. First I take the big 10-card. Which card shows how many ones are left over in 14 after we make 10? 4 I can put the 4-card over the zero on the 10-card. The 10-card is like a secret code telling us that 1 ten is hiding inside 14.



Emphasize the hidden 10 by drawing a dotted zero around the 4 in the 14 you had written on the board earlier.



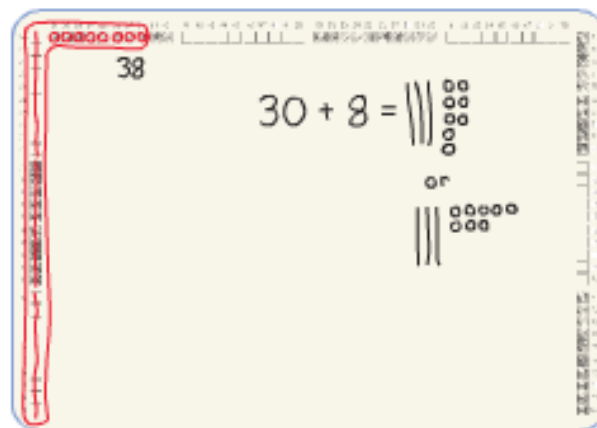
MAKE TENS AND ONES



- 10-sticks and circles
 - Easier to see tens and ones in a 2-digit number
 - Make it easier to understand the concept of regrouping when adding 2-digit numbers
- Word problems
 - Make sense of the connection to the equation and the drawing to see the ten and more ones as a 2-digit number
- Comparing
 - See the ten stick as a ten hiding inside a number

ATTEND TO PRECISION (MP)

- ⦿ Ask child where 38 is on the board
- ⦿ Be aware that some students may say just the last square is 38
- ⦿ Make sure students understand that the number 38 really means 38 things
 - All squares on the number path up to and including that square make the quantity 38



DRAWING NUMBERS

Tools

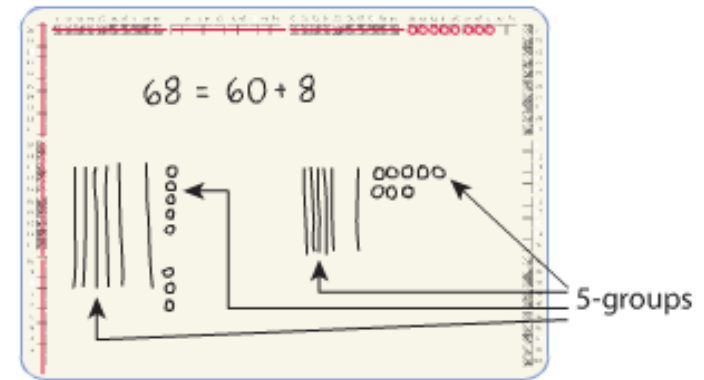
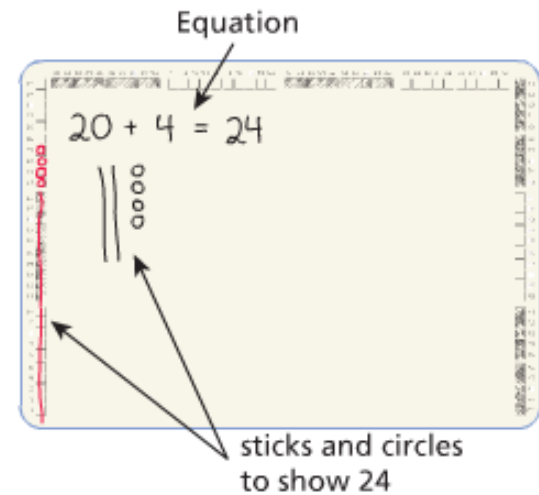
- Notice the number path
 - Identify number of tens and ones
- Write out equation
 - Draw sticks and circles
- Demonstrate with secret code cards

Precision

- Drawing with 5-groups

Structure

- Relationship between fast hundreds, quick tens, ones



LOOK FOR STRUCTURE (MP)

1	11	21	31	41	51	61	71	81	91	101	111
2	12	22	32	42	52	62	72	82	92	102	112
3	13	23	33	43	53	63	73	83	93	103	113
4	14	24	34	44	54	64	74	84	94	104	114
5	15	25	35	45	55	65	75	85	95	105	115
6	16	26	36	46	56	66	76	86	96	106	116
7	17	27	37	47	57	67	77	87	97	107	117
8	18	28	38	48	58	68	78	88	98	108	118
9	19	29	39	49	59	69	79	89	99	109	119
10	20	30	40	50	60	70	80	90	100	110	120

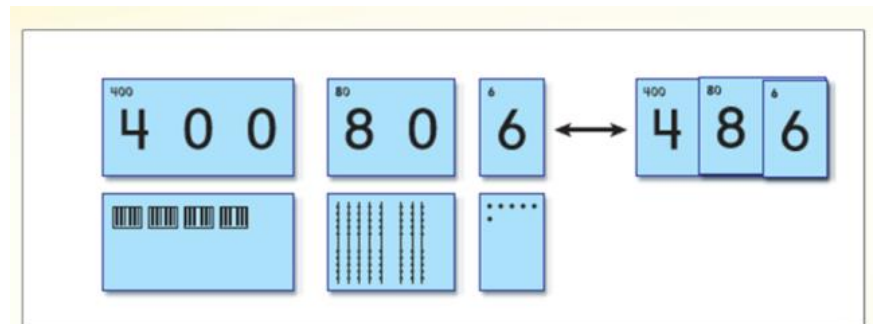
- What happens when 10 is added
 - Value shown in the tens place increases by 1 ten
- Look at $96 + 10$. What happens when you increase 96 by 1 ten?
 - We add 1 ten to 9 tens (and 6 ones)
 - $9 \text{ tens} + 1 \text{ ten} = 10 \text{ tens}$
 - 10 tens is the same as 1 hundred
 - So we have 1 hundred, 0 tens, and 6 ones
 - $100 + 0 + 6 = 106$

$$\begin{array}{ccc} 46 + 10 & 60 + 10 & 96 + 10 \\ 105 + 10 & 115 + 10 & \end{array}$$

SECRET CODE CARDS

Tools

- Standard form you read the English number and the position tells the value
 - See 486... just see single digits 4.8.6.
 - Only the location tells the value
- The cards show the “secret code” of our place value system
 - Expanded form gives the English repeating counting sequence
 - Four hundred eighty six
 - 4 hundreds 8 tens 6 ones
 - Layering
 - See the hidden zeros
 - Drawings
 - Show the values, 100 ones, or 10 tens



MATH TALK

Using secret code cards, drawings, equations as a tool to provide structure...

- The importance of the place value to explain how to find the expanded form of the number 562.

Digit x Place Value = Total Value

$$5 \times 100 = 500$$

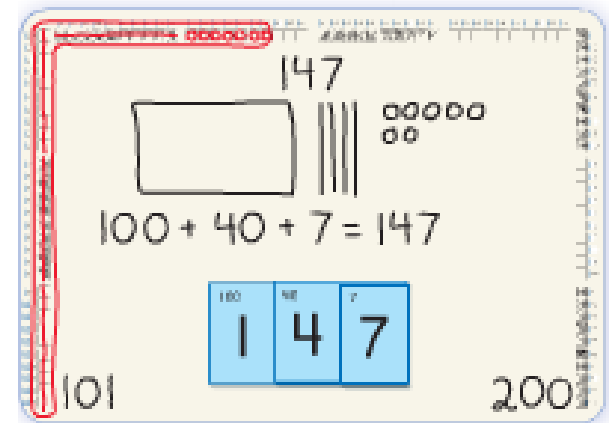
500 is the total value of 5

$$6 \times 10 = 60, \quad 2 \times 1 = 2$$

Combine total values $500 + 60 + 2$

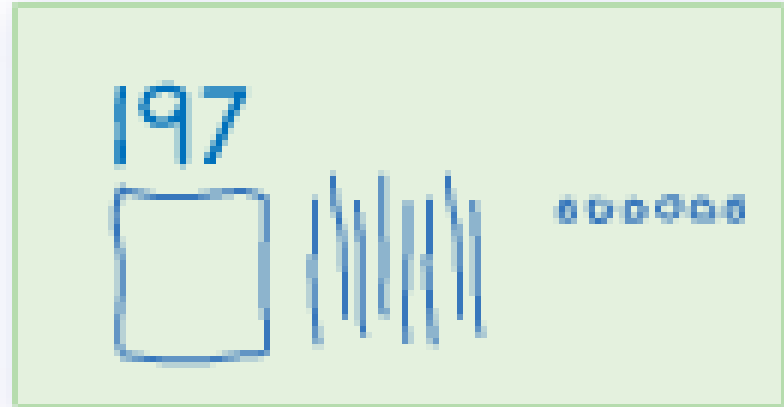
PLACE VALUE UNDERSTANDING

- Drawings
 - Helps see each number and be more accurate
- Expanded Form
 - Shows a number as the sum of the values of the digits in each place
- Secret Code Cards
 - Layered to see the place value of each digit and the value
- Numerals
 - Connect number names and then numerals and they read and write them



MATH TALK

- ⦿ What's the error?
- ⦿ What is the mistake?
- ⦿ What would have helped count the tens and ones correctly?



COMPARING

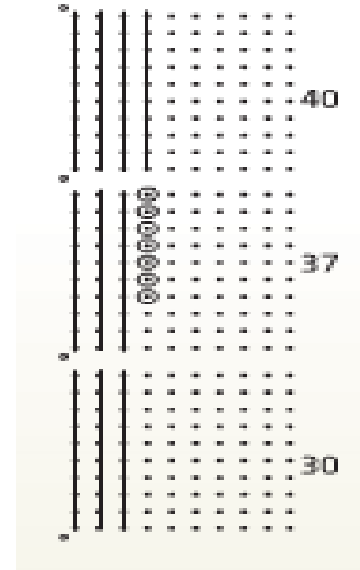
- Place value drawings connect understanding of place value

- Connects symbols drawings and digits



- Dot arrays drawings help visualize to which unit a number is closer

- Makes sense, position and value of the digit



- Students should know a 3 digit number can be rounded to the nearest thousand, hundred, or ten

- Round to the number with no ones (456-460)
- Round to number with no tens or ones (456-500)
 - Repeated reasoning, it works with ones, and tens...

REVIEW FROM KINDERGARTEN

- Sequencing and visualizing 5
 - Perceptual subitizing
 - Recognizing the cardinality of small groups without counting objects
 - Conceptual subitizing
 - Recognizing a collection is composed of 2 subcollections
- Focus on moving children from Count All to Count On
- Moving toward fluency with $+0/-0$ and $+1/-1$
 - Look for patterns
- Problem Solving
 - Informally solving put together/take apart by decomposing a number (later formal ways will be explored)

LEVELS OF ADDITION & SUBTRACTION SOLUTION METHODS

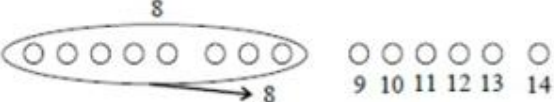
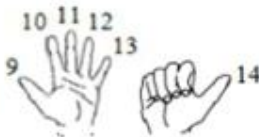
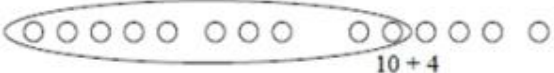
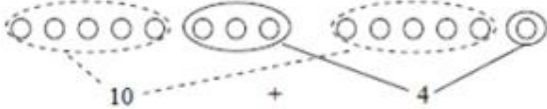
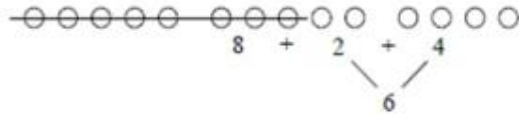
- Level 1- Count all
 - Model numbers, represent all of the objects
- Level 2-Counting on
 - Embed 1 addend in the total, to shorten the final count
 - Subtraction can be thought of as an unknown addend
- Level 3 -Make-a-ten
 - Recompose a given problem into an easier problem

Figure 4. Levels of Addition and Subtraction Solution Methods

<p>Addition Level 1 Count All</p> <p>○○○○○ ○○○○ ○○○</p> <p>$9 + 4 = 13$</p>	<p>Subtraction Level 1 Take Away</p> <p>○○○○○ ○○○○ ○○○</p> <p>$13 - 9 = 4$</p>
<p>Addition Level 2 Count On to Find a Total</p> <p>9 ○○○○</p> <p>Stop when counted on 4 more, 13</p> <p>$9 + 4 = 13$</p> <p style="text-align: center;"> $\begin{array}{c} 13 \\ \wedge \\ 9 \quad 4 \end{array}$ </p>	<p>Unknown Addend and Subtraction Level 2 Count On to Find an Unknown Addend</p> <p>9 ○○○○</p> <p>Stop when counted to 13, 4 more, so 4</p> <p>$9 + 4 = 13$ $13 - 9 = 4$</p> <p style="text-align: center;"> $\begin{array}{c} 13 \\ \wedge \\ 9 \quad 4 \end{array}$ </p>
<p>Addition Level 3 Derived Facts Make a Ten</p> <p>$9 + 4$ is $9 + 1 + 3 = 10 + 3$</p> <p>9 ○○○○</p> <p>$10 + 3 = 13$</p>	<p>Unknown Addend and Subtraction Level 3 Derived Facts Make a Ten</p> <p>$13 - 9 = \square$</p> <p>$9 + \square = 13$</p> <p>$9 + 1 + 3$ 4 more to make 13</p> <p style="text-align: center;"> $\begin{array}{c} \square \\ \vee \\ 4 \end{array}$ </p>

LEVELS OF ADDITION AND SUBTRACTION METHODS

Levels of Children's Addition and Subtraction Methods

Levels	$8 + 6 = 14$	$14 - 8 = 6$
Level 1: Count all	<p>Count All</p> <p>a</p> <p>1 2 3 4 5 6 7 8</p> <p>1 2 3 4 5 6</p> <p>c</p> <p>b</p> <p>1 2 3 4 5 6</p> <p>9 10 11 12 13 14</p>	<p>Take Away</p> <p>a</p> <p>1 2 3 4 5 6 7 8 9 10 11 12 13 14</p> <p>1 2 3 4 5 6 7 8 1 2 3 4 5 6</p> <p>b</p> <p>c</p>
Level 2: Count on	<p>Count On</p> <p>8</p>  <p>9 10 11 12 13 14</p>	<p>To solve $14 - 8$ I count on $8 + ? = 14$</p>  <p>I took away 8</p> <p>8 to 14 is 6 so $14 - 8 = 6$</p>
Level 3: Recompose	<p>Recompose: Make a Ten</p>  <p>10 + 4</p>  <p>10 + 4</p>	<p>$14 - 8$: I make a ten for $8 + ? = 14$</p>  <p>8 + 6 = 14</p>
Doubles $\pm n$	<p>$6 + 7$ $= 6 + 6 + 1$ $= 12 + 1 = 13$</p>	<p>$6 + ? = 13$ $= 6 + 6 + 1$ $= 12 + 1 = 13$</p>

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

CRITICAL CONCEPTS

- Embedded number concept is crucial!
 - Counting On!
 - Learning all the prerequisites in kindergarten for the making-a-ten

Concepts for Each Level

Level 1: Each new addend and the total are separate.

Level 2: Both addends are embedded within the total.

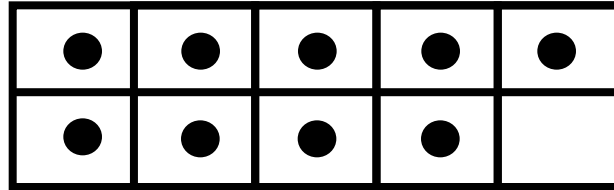
Level 3: Addends are recomposed to make new addends (e.g. $8+6$ becomes $10 + 4$)

Make-a-ten prerequisites

- Partner of the larger addend to 10 (K.OA.4)
- All partners of the smaller addend to find how much over ten (K.OA.3)
- $10 + n$ for $n = 1$ to 9 (K.NBT.1)

TEN-FRAME FLASH ACTIVITY

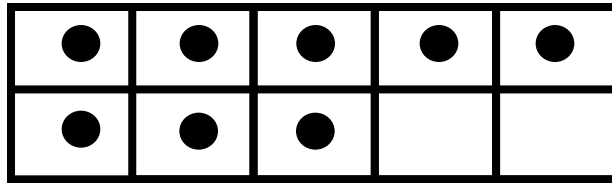
- Flash ten-frame cards to the class or group, and see how fast the children can tell how many dots are shown.



- Variations:
 - Saying the number of spaces on the card instead of the number of dots
 - Saying the number partners - for example, “Six and four make ten”

MAKE A TEN

Hold out a dot plate or ten frame showing 10 or less. Say, “How many more makes 10?”



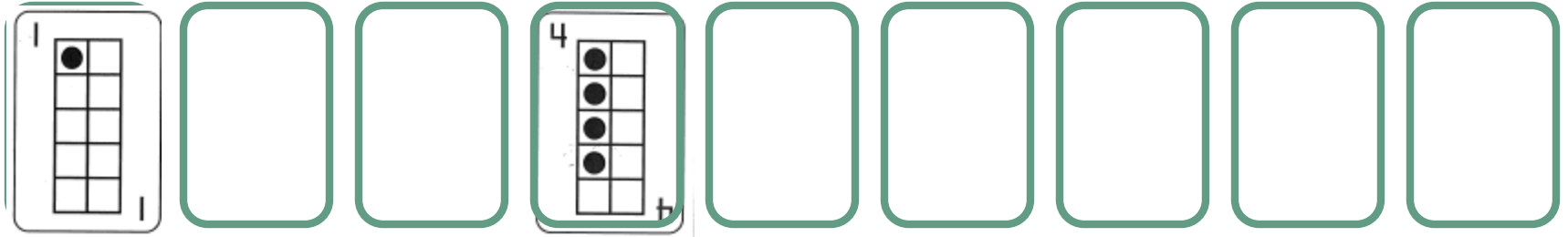
- The children respond with the part that is needed to make 10. Counting can be used to check.
- **Options:** Students can record the number families for making ten.

TEN MATCH UPS

Number of players: 2

Materials: Two sets of Ten Frame cards (1-9).

TEN MATCH UPS

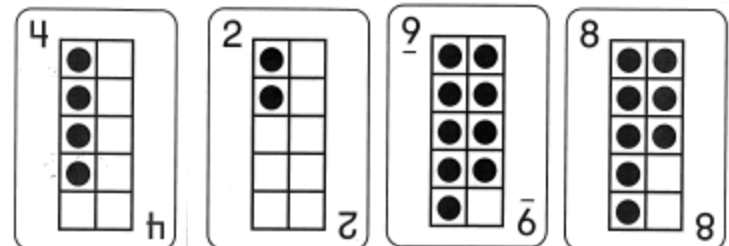
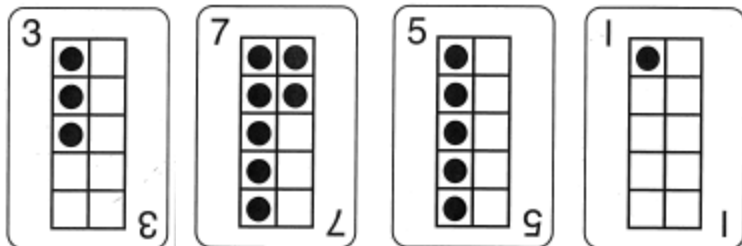


1. Put one set of ten frame cards face down in a row in the middle of the table.
2. Mix up the other set and hand them out so that each player gets 4 cards.
Put them in order face up in your own line.

3. Take turns. Take a ten frame card from the middle of the table and turn it face up.
4. If you have the ten partner, keep the picture card and place it face up, on top of the partner card and describe the match up. (6 and 4 makes 10.) If you do not have the ten partner, put the card back, face down, where you found it.
5. The first player to make all 4 matches wins the game.

Player 1

Player 2

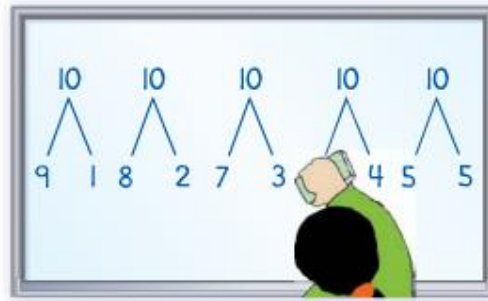


Let's Play!

"9 and 1 makes 10"

NUMBER GRABBER

- Write math mountains with totals of 10.



- Students cover their eyes.
- Have a student come up and erase a number partner.
- Say, “Open your eyes for a big surprise”
- Have a volunteer restore the missing unknown partner.

MORE SUPPORTING RESOURCES

- ◎ 10 Partner Fish
- ◎ 10 Partner Concentration
 - Variations (Small group or Individual)
 - Play with the cards face up
 - Begin face up, turn them over and play face down.
 - Begin face down and play.

SEEING TENS IN TEENS

Structure and Relationships

- Use other objects for students to build teen numbers
 - Centimeter cubes, counters, tiles
- Becoming more abstract
 - Partner expressions and teen number match up
 - Understanding the representation
 - This helps students conceptually see the chunking of the numbers

$10+1$	11
$10+2$	12
$10+3$	13
$10+4$	14
$10+5$	15

$10+6$	16
$10+7$	17
$10+8$	18
$10+9$	19

or

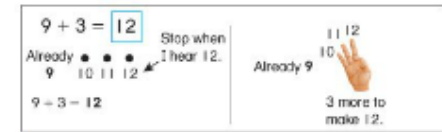
11	12	13	14	15
$10+1$	$10+2$	$10+3$	$10+4$	$10+5$
16	17	18	19	
$10+6$	$10+7$	$10+8$	$10+9$	

COUNTING ON FOR ADDITION & SUBTRACTION

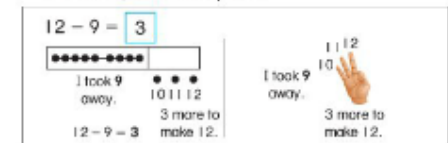
- Construct viable arguments to count on to find the total or addend (partner)

$$9 + 3 = 12$$

- In counting on to find the total 12, you keep track of the second addend 3 and stop when your fingers show 3. The words tell the total 12.



- In counting on to find the unknown addend 3, you keep track of the words you say and stop when you hear 12. The number of fingers tell the unknown addend 3.



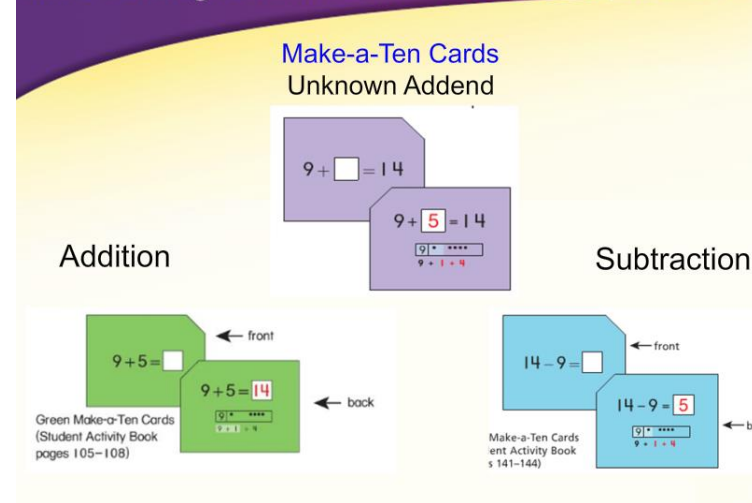
- Reason Abstractly and Quantitatively

- Connecting the fingers, drawings, and equations

FLUENCY

Make a ten strategy cards

- Grade K - within 5
 - Grade 1 - teen totals
 - Grade 2- fluency within 20
 - Support strategy use that have not yet finalized a strategy
- The first addend and how many make a ten, the rest of the second addend is separated with a space
- See $(9+1)$ oh $10 + 4$ is 14
- Beginning fluency
- Doing a very rapid strategy is more dependable than memorizing



FLUENCY WITHIN 20

- Work around the card

- Find total
- Find addend

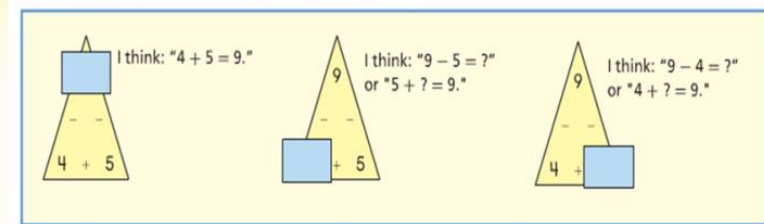
- Builds crucial relationships between addition and subtraction

- Important to start with sums less than or equal to 10

- All kids should know this from memory or by counting on

Grade 2 Fluency Within 20

Math Mountain Cards to practice relationships among the addends and the total



BUILDING FLUENCY

- Grades 1 & 2 Fluency within 20
- Grade 1 fluency within 10
- This plan drops the extra flash cards and uses Fast Friendly Go Around pages
- Fluency Checks should happen at the end of each “Big Idea”

- Please look at packet.

Contents and Order of the FFGA Practice Pages 9-21-3013

Step 1: The Basic Pattern Additions/Subtractions: $n+1/n-1, n+0/n-0$, doubles, $5+n$, and Partners of 10 (Begin in Unit 1)

- A. Addition with 0
- B. Subtraction with 0
- C. Addition with 1
- D. Subtraction with 1
- E. Addition with doubles, 5-groups, and partners of ten
- F. Subtraction with doubles, 5-groups, and partners of ten
- G. Addition with all problems mixed
- H. Subtraction with all problems mixed

Step 2: Counting on for the Other Additions within Ten: $+2$ s and $+3$'s (Begin in Unit 2)

- I. The $+2$ pattern: $n+2$ ($4+2, 5+2, 6+2, 7+2, 8+2$) and its commutative version, $2+n$.
- J. The $+3$ pattern: $n+3$ ($4+3, 5+3, 6+3, 7+3$) and its commutative version, $3+n$.
- K. Addition with $+2$ and $+3$ mixed, and all problems within ten mixed.


Step 3: Counting On for the Other Subtractions within Ten. (Begin in Unit 3)

- L. $9-7, 6, 5, 4, 3, 2$ and its unknown addend version ($7 + [] = 9, 6 + [] = 9$, etc.)
- M. $8-6, 5, 4, 3, 2$ and its unknown addend version ($6 + [] = 8, 5 + [] = 8$, etc.)
- N. $7-5, 4, 3, 2$ and its unknown addend version ($5 + [] = 7, 4 + [] = 7$, etc.)
- O. $6-4, 3, 2$ and its unknown addend version ($4 + [] = 6, 3 + [] = 6$, etc.)
- P. Mixed Subtractions within 10

MAKE-A-TEN

Grade 1 Make-a-Ten Adding

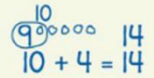
Use the Stair Steps



$8 + 6 = 10 + 4$

Use Math Drawings

$9 + 5$






10
 $9 + 5 = 14$
 $10 + 4 = 14$

- Grade 1 stair steps -manipulatives
- Drawings (counting on) so reconstruct the way you see count on with the drawings
 - See now $9+1$ more makes 10 and you know 10 and 4 makes 14
 - This drawing helps move to the mental
- Some students need to do this with multi-digit numbers to explain make-a-ten single digit because they can see the movement
 - Because breaking 2 digit sums $90+50$
 - 9 tens and 5 tens, that's breaks 9 tens and 1 ten to make 100 and 4 tens makes 140.

STUDENT TRANSITION STEPS

Not all students instantly master making-a-ten in two quick steps.

Some transition to this goal incrementally:

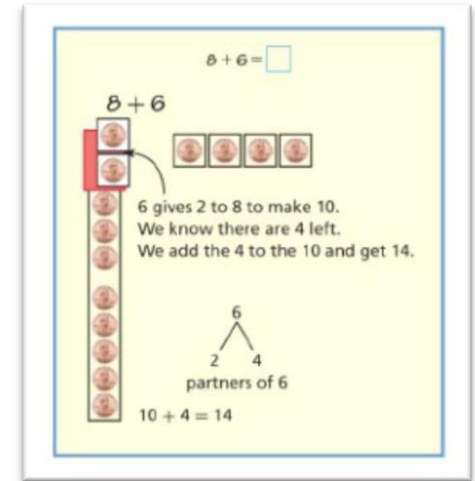
- 8 plus 2 is ten, 11, 12, 13, 14  **Add to 10... Count On the rest**
- 8, 9, 10 and 4 more is 14  **Count On to 10... Add the rest**
- 8 plus 2 is ten and 4 more is 14  **Add to 10... Add the rest**
- Use fingers
 - Hold up 6 fingers to represent the smaller addend.
 - Put down 2 to make ten.
 - 4 more makes 14.

COMMUTATIVE PROPERTY

Tools

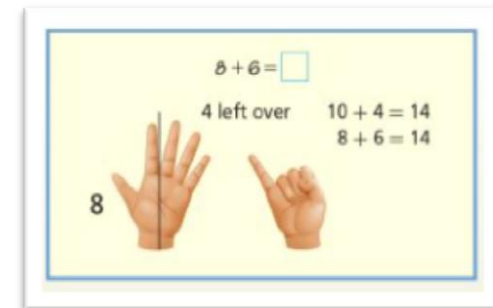
● Penny & Dime strips

- Summarize using math mountain to explain new problem (was $8+6$, now $10+4$)



● Fingers

- Separate the smaller addend into 2 parts
 - The amount added to the greater addend makes 10 and “the rest”



● Written

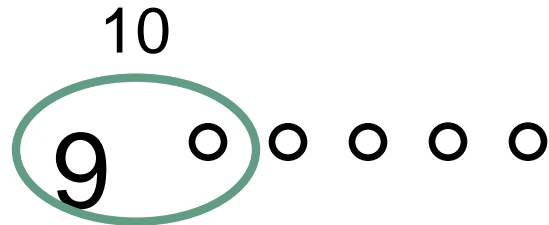
- Show various ways to make a ten

$8 \quad 00 0000$ $10 + 4 = 14$	8 $00 0000$ $10 + 4 = 14$	10 $00 0000$ $\begin{array}{r} 8 \\ + 6 \\ \hline 14 \end{array}$	$\begin{array}{r} + 8 \\ 6 \\ \hline 14 \end{array} \begin{array}{l} 2 \\ 4 \end{array}$
$8 + 6 = 14$ $\begin{array}{c} 8 \\ / \quad \backslash \\ 2 \quad 4 \end{array}$	$8 + 6 = 14$ $\begin{array}{c} 8 + 6 \\ / \quad \backslash \\ 10 \quad 2 \quad 4 \end{array}$	$\begin{array}{r} 8 \\ + 6 \\ \hline 14 \end{array} \begin{array}{l} 00 \\ / \quad \backslash \\ 4 \end{array}$	

REPRESENTATION EXPLORATION
OF THE
MAKE-A-TEN ADDITION STRATEGY

MAKE A TEN

- Draw a proof picture using the “Count-on” strategy to find the answer to $9 + 5$.



$$10 + 4 = 14$$

- Circle 10 in the drawing to isolate the 10 and some more.
- Label the 10 in the drawing.
- Write the “ten” equation.
- Now try and discuss: $7 + 5$ and $5 + 8$.

ABSTRACT EXPLORATION
OF THE
MAKE-A-TEN ADDITION STRATEGY

MATH MOUNTAINS

Student Writing for Understanding and Feedback

$$9 + 5 = \underline{14}$$

1 4

$$7 + 5 = \underline{12}$$

3 2

$$5 + 8 = \underline{13}$$

3 2

MAKE-A-TEN ADDITION

○ Introduce the Green Make-a-Ten Cards

- Look at the front and back of the $9 + 5$ card.
 - What do you think the dots on the back of the card show?
 - *They show 5.*
 - How are the 5 dots grouped?
 - *1 dot and 4 dots.*
 - What happens when you start with 9 and count on using the dot that is next to the 9?
 - *You get 10.*

MAKE-A-TEN ADDITION

- Introduce the Green Make-a-Ten Cards (Continued)
 - How can you use that 10 and the rest of the dots to get the answer?
 - *Count on from 10: 11, 12, 13, 14*
 - Is there a quicker way to get the answer?
 - *Add 9 and 1 and get 10, and then add 4 more and get 14.
10 + 4 is 14!*

MAKE-A-TEN ADDITION

Practice Finding Teen Totals

- Sort your cards according to the greater addend.
 - For example, find all the cards with 9 as the greater addend, then 8, then 7, then 6.
- Practice with each pile:
 - Begin with the 9 pile - *“1 more makes 10”*
 - then 8 - *“2 more makes 10”*
 - then 7 - *“3 more makes 10”*
 - then 6 - *“4 more makes 10”*.

MAKE A TEN FOR SUBTRACTION


- ◉ Same as addition, partner is unknown
- ◉ Problem situation and equations are different
- ◉ Looks complex but students only need to make 10 and add it to the amount over ten

$8 + \square = 14$

Step 1
Think: I already have 8.


Step 2
Put up 2 to make 10.

10
9



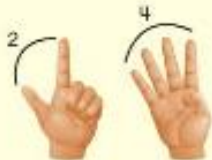
Step 3
Put up 4 more to make 14.

10 4 make 14



Step 4
Find the unknown partner.

2 4



$2 + 4 = 6$

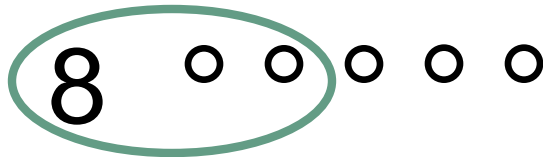
$$8 + \underline{\quad} = 14$$

MAKE-A-TEN PROOF DRAWINGS

- Draw a proof picture using the “Count-on” strategy to find the answer to $8 + \underline{\quad} = 13$.

2 more make 10

3 more make 13



$2 + 3 = 5$

5 is the unknown addend

- Circle 10 in the drawing to isolate the 10 and some more.
- Say how many more makes 10 and 13.
- Finally, add the parts together.
- Now try and discuss: $9 + \underline{\quad} = 15$ and $7 + \underline{\quad} = 12$.

RECORD FINDING THE MISSING ADDEND: COUNT-UP

Student Writing for Understanding and Feedback

$$9 + \frac{5}{\begin{array}{l} / \quad \backslash \\ +1 \quad +4 \end{array}} = 14$$

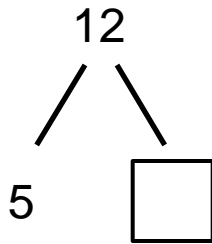
$$7 + \frac{4}{\begin{array}{l} / \quad \backslash \\ +3 \quad +1 \end{array}} = 11$$

$$8 + \frac{6}{\begin{array}{l} / \quad \backslash \\ +2 \quad +4 \end{array}} = 14$$

You Try!

THINK ADDITION TO SUBTRACT

- Look at the number mountain below.



- Write an addition and subtraction equation to help find the missing part.
 - $5 + \underline{\quad} = 12$ and $12 - 5 = \underline{\quad}$
- What is easier for most children to do, add or subtract?

MATH TALK: STRUCTURE

◎ Solution Process

Kim, how did you solve the first equation?

Kim: I made a ten and found out how many more up to the total.

$$9 + \square = 15$$

$$\begin{array}{r|l} 9 & 0 \ 0 \ 0 \ 0 \ 0 \\ 10 & 11 \ 12 \ 13 \ 14 \ 15 \end{array}$$

How did you solve the second equation?

Kim: I crossed out 9 and saw 1 more to make ten and 5 more, so my unknown addend is 6.

$$15 - 9 = \square$$

$$\begin{array}{c} \text{○○○○○} \text{ } \text{○○○○○} \text{ } \text{○○○○○} \\ \text{-----} \text{ } \text{-----} \text{ } \text{-----} \\ \text{○} \text{ } \text{○} \text{ } \text{○} \text{ } \text{○} \text{ } \text{○} \end{array}$$

How did you show that the solutions were the same?

Kim: I made a Math Mountain. A Math Mountain shows that you need to find a missing addend for both equations.



INTRODUCE THE PURPLE MAKE-A-TEN CARDS

- ⦿ Look through your set of Purple Make-a-Ten Cards and find “ $9 + \underline{\quad} = 13$ ”, “ $9 + \underline{\quad} = 14$ ”, “ $9 + \underline{\quad} = 15$ ”.
- ⦿ Look at both sides of each card and ask questions like the following:
 - How do you think the cards can be used to find the unknown partner?
 - What do you think the dots show on the cards?
 - Why do you think there are numbers below the dots?
 - Why do you think the cards are called Make-a-Ten Cards?

RECORD MAKE-A-TEN SUBTRACTION: COUNT-UP

Student Writing for Understanding and Feedback

$$12 - 9 = \underline{3}$$

/ \

+1 +2

$$11 - 7 = \underline{4}$$

/ \

+3 +1

$$13 - 8 = \underline{5}$$


/ \

+2 +3

You try!

DOUBLES

- Reasoning abstractly and quantitatively
 - How do you know when you can use the doubles plus 2 or minus 2 strategy to add 2 numbers?
 - Partners are 2 numbers apart
 - Ask how to show the doubles plus 2 or minus 2 strategy to find a sum.
 - Could be easier to use place value



1-7
Class Activity

Name _____

▶ Use Doubles

VOCABULARY

addition	subtraction
doubles	doubles
plus 1	minus 1
doubles plus 2	doubles minus 2

Addition Doubles and Subtraction Doubles	Doubles Plus 1 and Doubles Minus 1	Doubles Plus 2 and Doubles Minus 2
1. $5 + 5 = \boxed{10}$ $10 - 5 = \boxed{5}$	$5 + 6 = \boxed{11}$ $5 + 4 = \boxed{9}$	$5 + 7 = \boxed{12}$ $5 + 3 = \boxed{8}$
2. $6 + 6 = \boxed{12}$ $12 - 6 = \boxed{6}$	$6 + 7 = \boxed{13}$ $6 + 5 = \boxed{11}$	$6 + 8 = \boxed{14}$ $6 + 4 = \boxed{10}$
3. $7 + 7 = \boxed{14}$ $14 - 7 = \boxed{7}$	$7 + 8 = \boxed{15}$ $7 + 6 = \boxed{13}$	$7 + 9 = \boxed{16}$ $7 + 5 = \boxed{12}$
4. $8 + 8 = \boxed{16}$ $16 - 8 = \boxed{8}$	$8 + 9 = \boxed{17}$ $8 + 7 = \boxed{15}$	$8 + 10 = \boxed{18}$ $8 + 6 = \boxed{14}$
5. $9 + 9 = \boxed{18}$ $18 - 9 = \boxed{9}$	$9 + 10 = \boxed{19}$ $9 + 8 = \boxed{17}$	$9 + 11 = \boxed{20}$ $9 + 7 = \boxed{16}$

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UNIT 1 | LESSON 7
Strategies Using Doubles 21

NEW IN COMMON CORE STANDARDS

Operations and Algebraic Thinking

- ⦿ Solve problems with all 3 unknowns
- ⦿ Solve all problem types with all 3 unknowns
- ⦿ Each quantity can be unknown
- ⦿ Must show the situation with math drawing or diagram

WORD PROBLEMS

Grade 1 Relating Subtraction Representations

Class Activity Name _____

Solve and discuss.

1. We see 10 dogs.
7 run away.
How many are left?

$10 - 7 = \boxed{3}$

$7 + \boxed{3} = 10$

$\boxed{3}$ dogs label

2. We see 9 dogs.
5 are not barking.
The rest are barking.
How many are barking?

$9 - 5 = \boxed{4}$

$5 + \boxed{4} = 9$

$9 - \boxed{4} = 5$

$\boxed{4}$ dogs label

3. **Discuss** How are the methods you used to solve the problems alike and different?

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UNIT 3 LESSON 8 Practice with Subtraction Stories 81

Grade 1 Varying the Unknown for Addition

Class Activity Name _____

Solve and discuss.

1. There are 4 cats.
3 more cats join them.
How many cats are there now?

$4 + 3 = \boxed{7}$

2. There are 6 cats.
Some more cats join them.
Now there are 8 cats.
How many cats join?

$6 + \boxed{2} = 8$

3. There are some cats.
4 more cats join them.
Now there are 9 cats.
How many cats are there at the start?

$\boxed{5} + 4 = 9$

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UNIT 3 LESSON 9 Practice Addition and Subtraction Stories 83

Grade 1 Varying the Unknown for Subtraction

Class Activity Name _____

Solve and discuss.

4. There are 7 cats.
3 cats walk away.
How many cats are left?

$7 - 3 = \boxed{4}$

$3 + \boxed{4} = 7$

5. There are 8 cats.
Some cats walk away.
There are 6 cats left.
How many cats walk away?

$8 - \boxed{2} = 6$

$6 + \boxed{2} = 8$

6. There are some cats.
4 cats walk away.
Now there are 5 cats.
How many cats are there at the start?

$\boxed{9} - 4 = 5$

$5 + 4 = \boxed{9}$

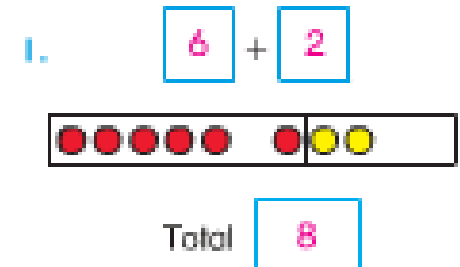
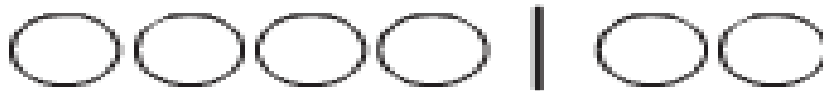
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UNIT 3 LESSON 9 Practice Addition and Subtraction Stories 84

CONCRETE TO REPRESENTATIONAL

○ Make sense and model

- Hands on with objects
- Replace pictures with simple drawings
 - Use stories to make sense of the simple drawings



○ Reason abstractly & Quantitatively

- Students find the partners to the picture
 - Connect the symbol (numeral) with drawing (object)
- Compare representation
 - Match pictures and simple drawings to “argue” most efficient way




REPRESENT ADDITION

Reason abstractly & Quantitatively

- Students find the partners to the picture
 - Connect the symbol (numeral) with model (object)
 - What partners are in this picture
 - 5 and 4
 - What total do you get when you add the flowers?
 - 9
 - How do you know the total is correct?
 - I counted the flowers and got 9

i.

$$\boxed{5} + \boxed{4}$$


Total $\boxed{9}$

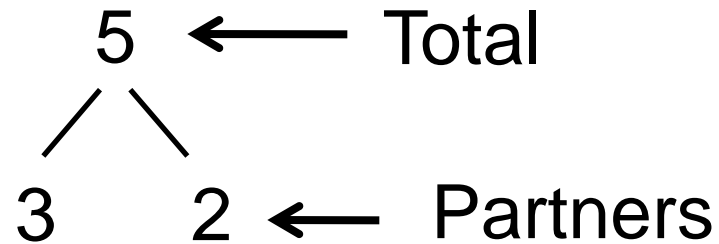
Check for understanding



Formative Assessment: Check Understanding

Student Summary Tell an addition math story and invite children to draw it. Invite them to describe their drawings using the terms *partners* and *total*.

MATH MOUNTAINS



- Shows the relationship among the addends (partners) and the total.
- Pretend there are 5 stones on top of this mountain. The mountain shakes and 2 stones roll down one side and 3 roll down the other side.
- Have students draw and identify the partners and total.

PROBLEM SOLVING PROCESS

- Understand the situation
 - Make sense of the language - to conceptualize the real world situation
 - Make sense of the problem
 - Reason Abstractly and quantitatively
- Represent the situation with a drawing/situation equation
 - Mathematize the situation - focus on mathematical aspects of situation
 - Model with mathematics
 - Look for and make use of structure
- Solve the representation (write a solution equation)
 - Find the answer - use drawings/situation/solution equation
 - Use appropriate tools
 - Use repeated reasoning
- Check the answer makes sense
 - Check the answer in the context of the problem - write and explain the label and answer
 - Critique the reasoning of others
 - Attend to precision






REPRESENTING THE SITUATION






Operations and **Algebraic thinking**

- ⦿ Represent the situation with a drawing, diagram and/or equation
 - A **situation** equation shows the action or the relationships in a problem
- ⦿ Then decide how to solve for the answer
 - A **solution** equation shows the operation that is performed to solve the problem

VIDEO: RELATING EQUATIONS

- Becoming flexible problem solvers
- Understanding the total and addends in the equation

	$9 + 3 = 12$		$12 = 9 + 3$
	$3 + 9 = 12$		$12 = 3 + 9$
	$12 - 9 = 3$		$3 = 12 - 9$
	$12 - 3 = 9$		$9 = 12 - 3$

	$9 + 3 = \underline{12}$		$\underline{12} = 9 + 3$
	$3 + 9 = \underline{12}$		$\underline{12} = 3 + 9$
	$\underline{12} - 9 = 3$		$3 = \underline{12} - 9$
	$\underline{12} - 3 = 9$		$9 = \underline{12} - 3$

EXPLORE TEEN EQUATIONS CARDS

- What is the difference between Set A and Set B?
- Discuss and review the meaning of the = and + signs.
 - For showing partners, the total has always been first.
 - For addition stories, the partners have been first.
 - We can write the equation either way. Both are correct.

Set A

$11 = 10 + 1$	$12 = 10 + 2$	$13 = 10 + 3$	$14 = 10 + 4$
$16 = 10 + 6$	$17 = 10 + 7$	$18 = 10 + 8$	$19 = 10 + 9$

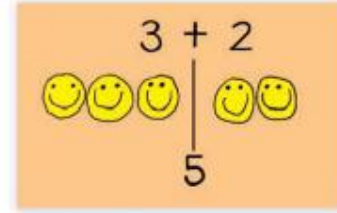
Set B

$10 + 1 = 11$	$10 + 2 = 12$	$10 + 3 = 13$	$10 + 4 = 14$
$10 + 6 = 16$	$10 + 7 = 17$	$10 + 8 = 18$	$10 + 9 = 19$

REPRESENT ADDITION

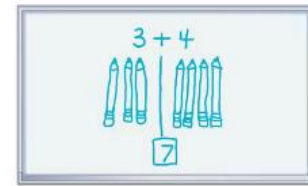
Make Sense of problems

- Draw a picture of the math story
- Draw a break apart line between the two groups of people



Storyatize

- Helps demonstrate mathematical concepts in a visual and memorable way



Model with Mathematics

- What are the partners in the math story?
- What is the total?

Mathematize

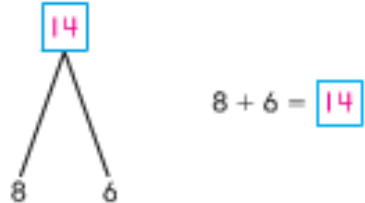
- Using standard notation from concrete to traditional equations

RELATING ADDITION AND SUBTRACTION

Math Mountains

- Connecting diagrams & equations
 - Partners (addends)
 - Total
- Word Problems
 - Use Structure to explain the situation
 - Partners (addends) used to find total
 - Use Precision to explain the strategy
 - Count all, Count on, Make-a-ten
 - Use Precision to explain if finding a partner or total
 - Add To / Take From
 - Put Together / Take Apart

▶ **Relate Math Mountains and Equations for Addition**

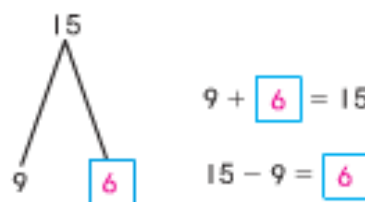


$8 + 6 = 14$

Discuss the **Math Mountain** and the **equation**.

1. Where is the **total**? Where are the **partners** or **addends**?
2. Tell word problems for both.
3. Solve both and compare your strategies.

▶ **Relate Math Mountains and Equations for Subtraction**



$9 + 6 = 15$
 $15 - 9 = 6$

Discuss this Math Mountain and the equations.

4. Where is the total? Where are the partners or addends?
5. Tell word problems for all.
6. Solve all and compare your strategies.

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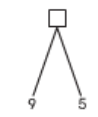
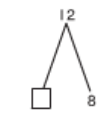
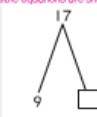
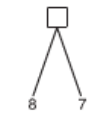
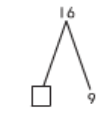
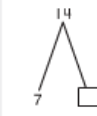
MATH TALK

Solve and discuss using precision

- Start with math drawing/diagram
 - Diagrams show relationship between drawings
- Diagram explains situation equations
 - Helps to see the first number in the subtraction equation is the total
 - See all 8 related equations
 - Connect addition and subtraction and their experience in various forms
 - Use word problems to make sure the equation can be used to solve that situation

► Write Equations for Math Mountains

10. Write two equations for each Math Mountain. Equations may vary. Possible equations are shown.

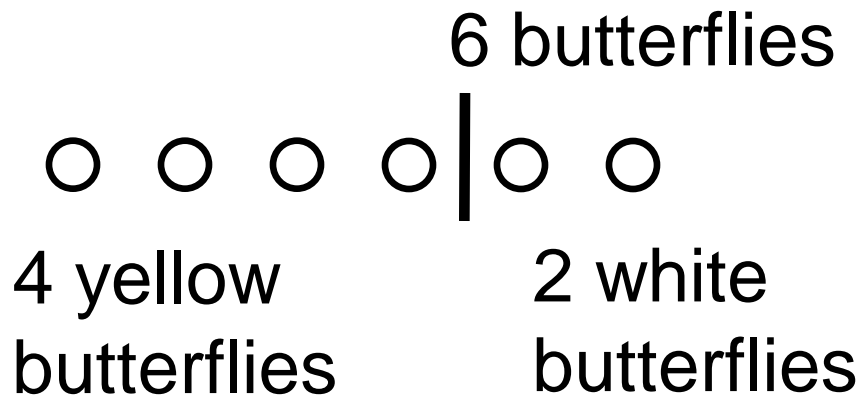
 $9 + 5 = \square$ $\square = 9 + 5$	 $12 - 8 = \square$ $8 + \square = 12$	 $9 + \square = 17$ $17 - 9 = \square$
 $8 + 7 = \square$ $\square = 7 + 8$	 $16 - 9 = \square$ $9 + \square = 16$	 $7 + \square = 14$ $14 - 7 = \square$

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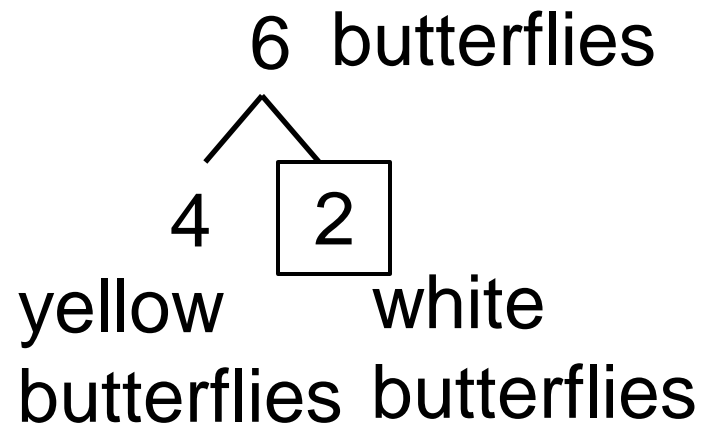
INTRODUCE AN UNKNOWN PARTNER

- These are the first addition situations that look for a missing partner.
- I see 6 butterflies. 4 are yellow. The rest are white. How many butterflies are white?

Circle Drawings



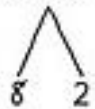
Math Mountains



YOU TRY!

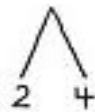
Draw another mountain and solve for the unknown total.

Find an unknown total. Draw the mountain shown below. Its total is missing. Have children draw the mountain at their desks and find the total.



- Look at this mountain. Let's draw it. What number belongs on the top of this mountain? **10** How do you know? **The partners are 8 and 2. I counted on to find that the total is 10.**

Let's try another.



- What about this mountain? What is the unknown number? **6**
- How do you know? **6 is the total of the partners 2 and 4.**

Draw another mountain and solve for the unknown partner.

Find an unknown partner. Draw the mountain shown below and have children copy it. This Math Mountain has one unknown partner.



- The total is 7. If 4 stones roll down this side of the mountain, how can we find out the number that rolls down the other side?

Explain that one way to find the unknown partner is to count on. Encourage children to use mental math by keeping the known partner, 4, "in their mind" as they begin. Then they draw one circle for each number they count on. Note that the numbers 5, 6, and 7 are not written. They are only spoken as each circle is drawn.

- For every number I count on, I will draw a circle. Let's count on from 4 until we get to the total, 7. Count with me: 5, 6, 7.



Have children use this method by drawing circles on their mountains as you lead them again in counting on to 7. Children are using mental math as they learn to count on from 4 to find the unknown partner.

- How many circles did you draw? **3**

Identify 3 as the unknown partner and write it on the mountain.

WORD PROBLEM SITUATIONS

Drawings best for smaller numbers

Diagrams to show the relationship of the numbers

Problem Type	Word Problem	Representation	
		Math Drawing	Diagram
Add To	Dan had 9 cherries. Then he picked 4 more. How many does he have now?		$9+4=\square$ (situation/solution equation)
Take From	Dan had 13 cherries. Then he ate 9 of them. How many does he have now?		$13-9=\square$ (situation/solution equation)
Put Together/ Take Apart	Ana has 9 dimes and 4 nickels. How many coins does she have in all?		Math Mountain Diagram
Additive Comparison	Ali has 9 balloons. Lisa has 13 balloons. How many more balloons does Lisa have than Ali?	Matching Drawing 	Comparison Bars
Equal Groups	Amy has 5 cousins. She is making 2 puppets for each cousin. How many puppets will Amy need to make?	Grouping Model 	Equal Shares Diagram
Array	A garden has 5 rows and 2 columns of bean plants. How many plants are there in all?	Array Model 	Fast Array Diagram
Area	The garden is 5 yards on one side and 2 yards on the side touching this. What is its area?	Area Model 	Fast Area Diagram
Multiplicative Comparison	Bill has 2 apples. Kim has 5 times as many apples as Bill. How many apples does Kim have?	Grouping Model 	Comparison Bars $B=K+5$

ADDITION AND SUBTRACTION SITUATIONS

○ Add To/Take From

- Result unknown
- Change unknown
- Start unknown

○ Put Together/Take Apart

- Total unknown
- Both addends unknown
- Addend unknown

○ Compare

- Difference unknown
- Bigger unknown
- Smaller unknown

Table 2. Addition and subtraction situations by grade level.

	Result Unknown	Change Unknown	Start Unknown
Add To	<p>A bunnies sat on the grass. B more bunnies hopped there. How many bunnies are on the grass now?</p> $A + B = \square$	<p>A bunnies were sitting on the grass. Some more bunnies hopped there. Then there were C bunnies. How many bunnies hopped over to the first two?</p> $A + \square = C$	<p>Some bunnies were sitting on the grass. B more bunnies hopped there. Then there were C bunnies. How many bunnies were on the grass before?</p> $\square + B = C$
Take From	<p>C apples were on the table. I ate B apples. How many apples are on the table now?</p> $C - B = \square$	<p>C apples were on the table. I ate some apples. Then there were A apples. How many apples did I eat?</p> $C - \square = A$	<p>Some apples were on the table. I ate B apples. Then there were A apples. How many apples were on the table before?</p> $\square - B = A$
Put Together/Take Apart	<p>Total Unknown</p> <p>A red apples and B green apples are on the table. How many apples are on the table?</p> $A + B = \square$	<p>Both Addends Unknown²</p> <p>Grandma has C flowers. How many can she put in her red vase and how many in her blue vase?</p> $C = \square + \square$	<p>Addend Unknown²</p> <p>C apples are on the table. A are red and the rest are green. How many apples are green?</p> $A + \square = C$ $C - A = \square$
Compare	<p>Difference Unknown</p> <p>"How many more?" version. Lucy has A apples. Julie has C apples. How many more apples does Julie have than Lucy?</p> <p>"How many fewer?" version. Lucy has A apples. Julie has C apples. How many fewer apples does Lucy have than Julie?</p> $A + \square = C$ $C - A = \square$	<p>Bigger Unknown</p> <p>"More" version suggests operation. Julie has B more apples than Lucy. Lucy has A apples. How many apples does Julie have?</p> <p>"Fewer" version suggests wrong operation. Lucy has B fewer apples than Julie. Lucy has A apples. How many apples does Julie have?</p> $A + B = \square$	<p>Smaller Unknown</p> <p>"Fewer" version suggests operation. Lucy has B fewer apples than Julie. Julie has C apples. How many apples does Lucy have?</p> <p>"More" version suggests wrong operation. Julie has B more apples than Lucy. Julie has C apples. How many apples does Lucy have?</p> $C - B = \square$ $\square + B = C$

LABEL MATH DRAWINGS

- Key is understanding the situation
- Labeling explains the parts of the story

Grade 2 Labeled Math Drawings for a Start Unknown Problem

Yolanda has a box of golf balls. Eddie took 7 of them. Now Yolanda has 5 left. How many golf balls did Yolanda have in the beginning?

The key to solving story problems is **understanding the situation**. Students' equations often show the **situation** rather than the solution. Students drawings should be **labeled** to show which numbers or objects show which parts of the story situation.

The student's work is divided into three sections:

- Left Section:** Shows a subtraction equation: $\boxed{12} - 7 = 5$. The number 12 is labeled "Beginning" and "golf balls label". The number 7 is labeled "Yol E" (Yolanda took 7). The number 5 is labeled "Total". To the right is a drawing of a box with 12 golf balls inside, with "12" written below it.
- Right Section:** Shows a tree diagram. The top node is $\boxed{12}$ labeled "Y in Beginning" and "golf balls label". Two lines branch down to 7 and 5. Below the diagram is the text "I put the golf balls back together."
- Bottom Section:** Shows an addition equation: $\begin{array}{r} 7 \text{ E} \\ + 5 \text{ Yolanda} \\ \hline 12 \end{array}$. Below this is the text "12 golf ball" with "12" in a box and "golf ball" labeled.

LABEL VS. TOTAL

Why do students answer 13 bees?

First grader thinking:

- First he drew a mountain with 8 & 5 as partners
- Next he wrote 13
- Then he circled bees in the first sentence and bees in question
- Finally he wrote bees

Multiple ways to answer

- Math sense making
 - Real world connection
- Pictures, diagrams, equations
 - Labeling partners/total
 - Label for understanding
 - Label for the unit

3-2

Name _____

Homework

Solve the story problem.

1. I see 8 bees in the sky,
5 fly low. The others fly high.
How many bees fly high?

3

bees

label

2. 7 cars are in the parking lot.
Then more cars come.
Now there are 9.
How many more cars come?

2

more cars

label

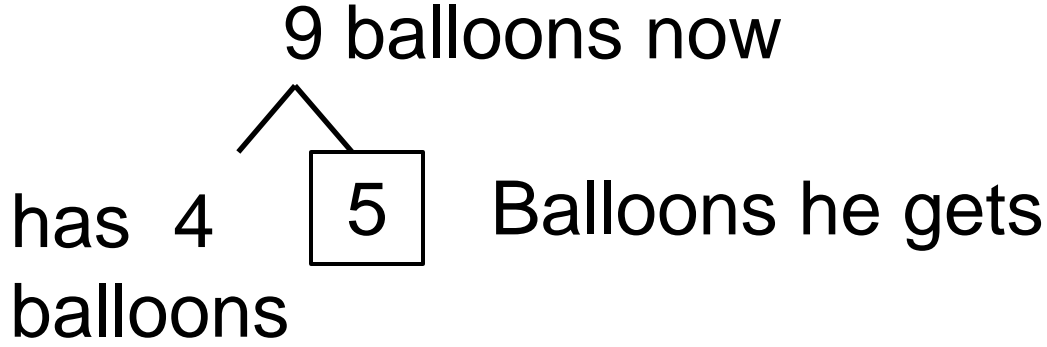
SUBTRACTION AS UNKNOWN PARTNER

Balloons

- Sam has 4 balloons. Then he ~~gets some more~~. Now he has 9. How many balloons does he get?

Balloons

Math Mountains

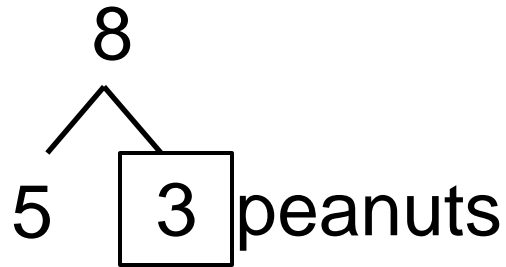


5 Balloons
label

SUBTRACTION AS UNKNOWN PARTNER

- I have 8 peanuts. Then I eat 5 of them. How many peanuts are left?

Math Mountains



- Solve by counting on from 5.
- Read story again. Is it plus or minus?



8 peanuts I eat 5 peanuts

- Write the equation.

$$8 - 5 = \boxed{3}$$

REPRESENT SUBTRACTION

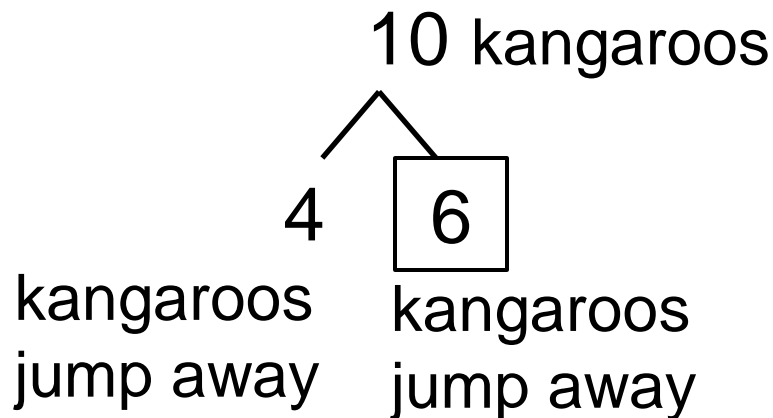
We see 10 kangaroos.

Then 4 of them jump away.

How many kangaroos are left?

○ Have students draw a math mountain

- Which number is the total
- Which are the partners
- Write a subtraction equation that shows this story
- Where is the total now?
- Where are the partners?
- Write an addition equation that shows this story.



$$10 - 4 = \boxed{6}$$

$$10 = 4 + \boxed{6}$$

DISCUSS WITH YOUR TABLE

- Why is this set up the way it is?
- Why might some people think this is confusing...
- Understanding Equations:
 - Situation
 - Solution

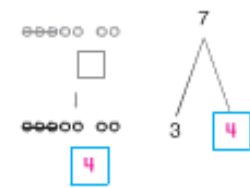
Class Activity Name _____

Solve and discuss.

4. There are 7 cats.
3 cats walk away.
How many cats are left?

$7 - 3 = 4$

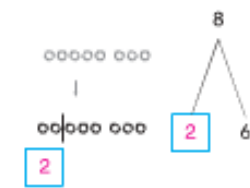
$3 + 4 = 7$



5. There are 8 cats.
Some cats walk away.
There are 6 cats left.
How many cats walk away?

$8 - 2 = 6$

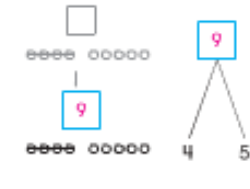
$6 + 2 = 8$



6. There are some cats.
4 cats walk away.
Now there are 5 cats.
How many cats are there
all the start?

$9 - 4 = 5$

$5 + 4 = 9$



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Math Addition and Subtraction Equations

ONE STEP WORD PROBLEM

Shayna had some markers. She gave 5 of the markers to her friends. Now she has 2 markers. How many markers did she have in the beginning?

- How many markers did Shayna give away?
 - 5
- How many markers did Shayna have left?
 - 2
- What are you trying to find out?
 - The number of markers Shayna had when she started
- Is this the unknown number in the situation?
 - Yes
- How can we find the unknown number or solution?
 - Add $5+2$ (the number of markers that Shayna gave away plus the number of markers that were left)
- Situation: $b - 5 = 2$ Solution: $b = 2 + 5$

REPRESENT WORD PROBLEMS

- Do you see the difference now about situation and solution?

- Make sense of problems
 - Circle drawings and Math Mountains
 - A solution equation is an equation that helps solve the problem
 - Show addition and subtraction equation are related because they use the same numbers, but they are opposite operations
 - Any of the numbers can be an unknown in a problem

1-4 Class Activity

Name _____

Solve and discuss.

4. There are 7 cats.
3 cats walk away.
How many cats are left?

$7 - 3 = \boxed{4}$

$3 + \boxed{4} = 7$

5. There are 8 cats.
Some cats walk away.
There are 6 cats left.
How many cats walk away?

$8 - \boxed{2} = 6$

$6 + \boxed{2} = 8$

6. There are some cats.
4 cats walk away.
Now there are 5 cats.
How many cats are there at the start?

$\boxed{9} - 4 = 5$

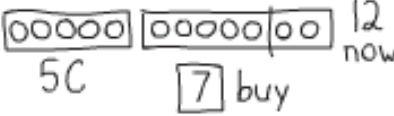
$5 + 4 = \boxed{9}$

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Prints Addition and Subtraction Equations

REPRESENT WORD PROBLEMS

- Make sense of problems
 - Ways to rephrase
 - The school starts with 5 computers. Then the school buys some more computers. They have 12 computers now. How many more computers does the school buy?
 - Label all parts of the problem

$5 + \boxed{7} = 12$ c buy now	 5C $\boxed{7}$ buy 12 now
$5 + \underbrace{5 + 2}_{\text{buy}} = 12$ c buy now	<div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> now 12 / \ 5 $\boxed{7}$ c buy </div> <div> $5 + \underbrace{00000}_{\text{more}} \underbrace{00}_{\text{now}} = 12$ C $\boxed{7}$ </div> </div>

1-10
Class Activity


Name _____


► Solve and Discuss

Make a drawing. Write an equation. Show your work.
 Solve the problem. Children's drawings or equations may vary.

1. The school has 5 computers in the library. They buy some more computers. Now there are 12. How many computers does the school buy?


7
computers
label




 school

2. Alina has 17 beads. She uses 9 of them to make a bracelet. How many beads does she have left?


8
beads
label


 beads

17 beads - 9 used = 8 left
 $17 - 9 = \boxed{8}$

3. Erin wraps 6 party favors. She needs to wrap 15 favors in all. How many favors does she still need to wrap?

9
favors
label


 party favors


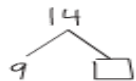
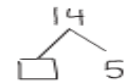
15 in all
 $6 + \underbrace{9}_{\text{more to wrap}} = 15$
 $6 + \boxed{9} = 15$

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UNIT 1 LESSON 10
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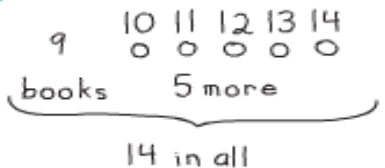
INTERPRET, REPRESENT, SOLVE, CHECK

- Retelling a story
 - Builds understanding
- Connects symbols and words (add to)
 - Vary position of the unknown and match equation, math mountain and word problem

Situation Equations and Math Mountains

Result Unknown	Change Unknown	Start Unknown
$9 + 5 = \square$ 	$9 + \square = 14$ 	$\square + 5 = 14$ 
Kevin reads 9 books. Then he reads 5 new books. How many books does Kevin read in all?	Kevin reads 9 books. Then he reads some new books. He reads 14 books in all. How many new books does Kevin read?	Kevin reads some books. Then he reads 5 new books. He reads 14 books in all. How many books does Kevin read at first (in the beginning, to start)?

- Draw a diagram/drawing
 - Use objects, objects and numbers, all numbers

	$10 + 4 = 14$ in all 9 start $\underbrace{0 \quad 4}_{5 \text{ new}}$	$\overbrace{5 + 5 + 4}^{14 \text{ in all}}$ new 9 to start
--------------------------------------------------------------------------------------	--------------------------------------------------------------------------	---------------------------------------------------------------

MATH TALK

- Solve and discuss using...
 - Drawings/diagrams
 - Math mountain
 - Equations

- Brian has some tomato plants in his garden. 9 of the plants are eaten by bugs. 4 plants are left. How many plants did Brian have in the beginning?

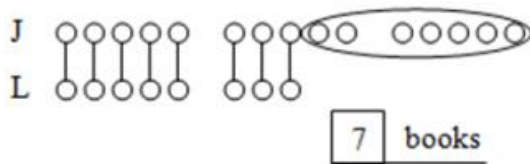
SOLUTION APPROACHES

Comparison problems

Figure 4. Grade 2 Solution Approaches to an Additive Comparison Problem

In March Jana read 15 books. Lisa read 8 books. How many fewer books did Lisa read than Jana?

Matching Drawing of Quantities

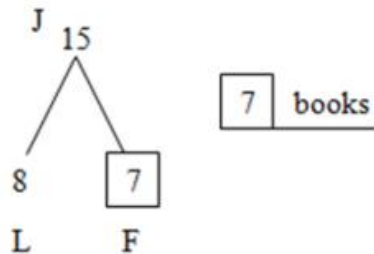


Situation Equation

$$8 + \boxed{7} = 15 \quad \boxed{7} \text{ books}$$

Lisa more Jana

Numerical Relationships Shown in Math Mountain



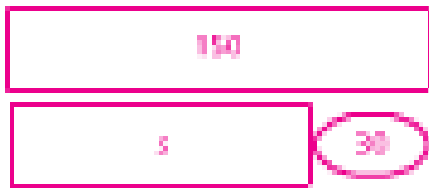
Solution Equation

$$15 - 8 = \boxed{7} \quad \boxed{7} \text{ books}$$

Jana Lisa fewer

ADDITION COMPARISON

- Read the problem below.
- The word more might lead students to believe that they should add, but they actually need to subtract
 - They know the greater quantity and the difference, so subtraction will give them the lesser quantity
- What took more time? How much more time?
- How do we show this?
- How do comparison bars help?



The soccer team **practiced** drilled for 150 minutes last week. The team drilled for 30 minutes more than it scrimmaged. For how long did the team **game** scrimmage? **practiced** **game**

MULTI-STEP PROBLEMS

- ⦿ Multiple entry points
- ⦿ More than one operation
- ⦿ Identify the helping question(s) or the question(s) that needs to be answered before the final solution can be found

WORD PROBLEMS

- Writing the first step equations
- Represent all steps

Write first-step equations

How many fruit on each plate?

Zoe made some snacks. She put 4 apple slices and 2 melon slices on each plate. She prepared 5 plates. How many slices of fruits did Zoe use in all?

total

How many slices of fruit are on each plate? 6

$(4 + 2) \times 5 = 30$; 30 slices of fruit in all

Or two separate steps with equations:

$$4 + 2 = 6 \quad \text{and} \quad 5 \times 6 = 30$$

Representing 2-Step and Multistep Problems

Students may write a single equation for some problems.

Mr. Helms has 2 stables with 4 horses in each stable. Ms. Martinez has 4 more horses than Mr. Helms. How many horses does Ms. Martinez have?

$$2 \times 4 + 4 = n, n = 12; 12 \text{ horses}$$

Some problems may require two steps of representation and solution or students may make drawings rather than equations.

Tim has 9 marbles. Ryan has 3 fewer marbles than Tim. Leslie has 5 more marbles than Ryan. How many marbles does Leslie have?

As always, students may represent or solve in different ways.

TWO STEP WORD PROBLEMS

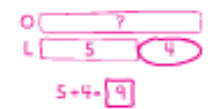
- Pay attention to the situation expressed in the problem
 - Not numbers and words/phrases
- What information does the problem ask for?
 - The number of cans Matt brings
- Paraphrase in their own words
- What is the hidden question?
 - How many cans does Olivia bring?
- Paraphrase again.

How many cans does Olivia bring?

1. ~~Lindsay brings in 5 cans for the school food drive. Olivia brings in 4 more cans than Lindsay. Matt brings in 6 more cans than Olivia. How many cans does Matt bring?~~

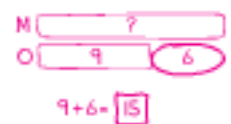
Olivia's 9 cans.

A. How many cans does Olivia bring?



9 _____ cans
label

B. How many cans does Matt bring?



15 _____ cans
label

MATH TALK

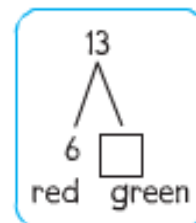
● Solve and discuss

- Rafa has 13 marbles. He has 6 red marbles and the rest are green. Mia gives him some more green marbles. Now Rafa has 12 green marbles. How many green marbles does Mia give Rafa?

Malek: We need to find how many green marbles Mia gave Rafa. So, we have to find out how many green marbles Rafa started with.

Sammy: The first-step question is: How many green marbles did Rafa start with?

Linda: We can use a Math Mountain to find out how many green marbles Rafa starts with.



Monique: Or you can use an equation to find the answer to the first-step question.

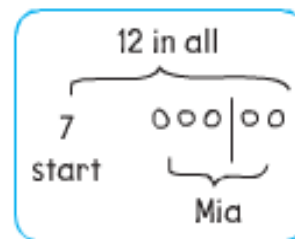
$$\begin{array}{c} 6 \\ \text{red} \end{array} + \begin{array}{c} \square \\ \text{green} \end{array} = 13 \text{ in all}$$

Dave: Then the answer to the problem is 7?

Gina: No, it means that Rafa had 7 green marbles to start with. Then Mia gave him some more green marbles and now he has 12 green marbles.

Dave: Got it. Now we have to find out how many Mia gave Rafa.

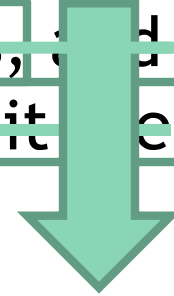
Max: We can use a drawing to find out how many green marbles Mia gave Rafa.



Nanette: So, Mia gave Rafa 5 green marbles.

MULTI-STEP WORD PROBLEMS

Isabel brought 36 pieces of fruit for her soccer team. There are 16 apples, 12 bananas, and the 28 fruit rest are pears. How many pieces of fruit are pears?



How many apples and bananas?
 $16 + 12$

What is the problem about?
Different kinds and amounts of fruit

What do you need to find?
The pieces of fruit that are pears

What do you know?
 $16 + 12 = 28$ fruit

Mathematize
Rephrase into your own words

Pay attention to the situation expressed in the problem
Not numbers and words/phrases

Situation: $36 = 28 + \underline{\text{pears}}$
Solve: $36 - 28 = 8$ pears

PROBLEM SOLVING PROCESS

The Problem Solving Process

Part A: Understand and represent: Conceptualize bottom up from the situation

Part B: Re-represent and solve: Use related problem types, representations, properties, and /or relationships between + - or \times

A1. Understand the problem situation

Mathematize (and Storyize)

A2. Represent the problem situation in a drawing/diagram and/or an equation

Then focus on the question and:

B1. Re-represent to find the unknown

Do the solution actions

B2. Write the answer and check that it makes sense

**Questions?
Comments...**

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